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Extensions and the irreducibilities of induced characters of some 2-groups

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Abstract. Let Q_n and D_n denote the generalized quaternion group and the dihedral group of order 2^{n+1} $(n \geq 2)$, respectively. Let SD_n denote the semidihedral group of order 2^{n+1} $(n \geq 3)$. Let ϕ be a faithful irreducible character of H, where $H = Q_n$ or D_n or SD_n. The purpose of this paper is to determine all 2-groups G such that $H \subset G$ and the induced character ϕ^G is also irreducible.

Key words: 2-group, induced character, faithful irreducible character, group extension.

1. Introduction

Let Q_n and D_n denote the generalized quaternion group and the dihedral group of order 2^{n+1} $(n \geq 2)$, respectively. Let SD_n denote the semidihedral group of order 2^{n+1} $(n \geq 3)$.

As is stated in [4], these groups have remarkable properties among all 2-groups.

Moreover, Yamada and Iida [5] proved the following interesting result: Let $\mathbf{Q}$ denote the rational field. Let G be a 2-group and χ a complex irreducible character of G. Then there exist subgroups $H \rhd N$ in G and the complex irreducible character ϕ of H such that $\chi = \phi^G$, $\mathbf{Q}(\chi) = \mathbf{Q}(\phi)$, $N = \operatorname{Ker}\phi$ and

$$H/N \cong Q_n\ (n \geq 2),\quad \text{or}\quad D_n\ (n \geq 3),\quad \text{or}\quad SD_n\ (n \geq 3),$$
$$\text{or}\quad C_n\ (n \geq 0),$$

where C_n is the cyclic group of order 2^n, and $\mathbf{Q}(\chi) = \mathbf{Q}(\chi(g),\ g \in G)$.

In [4], Yamada and Iida considered the case when $N = 1$, or equivalently ϕ is faithful. They studied the following problem:

Problem *Let ϕ be a faithful irreducible character of H, where $H = Q_n$ or D_n or SD_n. Determine the 2-group G such that $H \subset G$ and the induced character ϕ^G is also irreducible.*

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