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Certain sufficient conditions for univalence

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Abstract. In this work some integral operators are studied and the author determines conditions for the univalence of these integral operators.

Key words: integral operator, univalence.

1. Introduction

Let $U = \{z : |z| < 1\}$ be the unit disk in the complex plane and let A be the class of functions which are analytic in the unit disk normalized with $f(0) = f'(0) - 1 = 0$.

Let S the class of the functions $f \in A$ which are univalent in U.

2. Preliminary results

In order to prove our main results we will use the theorems presented in this section.

Theorem A [2] *Assume that $f \in A$ satisfies condition*

$$\left|\frac{z^2 f'(z)}{f^2(z)} - 1\right| < 1, \quad z \in U, \tag{1}$$

then f is univalent in U.

Theorem B [3] *Let α be a complex number,* $\operatorname{Re}\alpha > 0$ *and $f(z) = z + a_2 z^2 + \cdots$ is a regular function in U. If*

$$\frac{1 - |z|^{2\operatorname{Re}\alpha}}{\operatorname{Re}\alpha}\left|\frac{zf''(z)}{f'(z)}\right| \leq 1, \tag{2}$$

for all $z \in U$, then for any complex number β, $\operatorname{Re}\beta \geq \operatorname{Re}\alpha$ *the function*

$$F_\beta(z) = \left[\beta \int_o^z u^{\beta-1} f'(u)\,du\right]^{\frac{1}{\beta}} = z + \cdots \tag{3}$$

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