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Aluthge transformations and invariant subspaces of p-hyponormal operators

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Abstract. It is unknown at present whether every hyponormal operator has a nontrivial invariant subspace. Many authors presented conditions for a hyponormal operator to have nontrivial invariant subspaces. In this paper, we give a p-hyponormal version of Nakamura's result [7] by using the principal functions.

Key words: hyponormal operator, p-hyponormal operator, invariant subspace.

1. Introduction

An (bounded linear) operator T on a Hilbert space $\mathcal{H}$ is said to be p-hyponormal, if $(TT^*)^p \leq (T^*T)^p$ for a positive number p. If $p = 1$, then T is said to be hyponormal, and if $p = \frac{1}{2}$, then T is said to be semi-hyponormal. We assume that $0 < p \leq \frac{1}{2}$. An operator T is called pure if it has no nontrivial reducing subspace on which it is normal.

It is unknown at present whether every hyponormal operator has a nontrivial invariant subspace. Putnam [8] and Apostol and Clancey [2] presented some conditions for a hyponormal operator to have invariant subspaces. Nakamura [7] improved these results. In this paper, we give a p-hyponormal version of Nakamura's result.

Let $T = X + iY$ be a pure hyponormal operator, where X and Y are self-adjoint. Then it is known that X and Y are absolutely continuous (see [4, Chap. 2, Th. 3.2]). For a self-adjoint operator Z, let $Z = \int t dG(t)$ be the spectral resolution of Z. Then the absolutely continuous support E_Z of Z is defined as a Borel subset of the real line (determined uniquely up to a null set) having the least Lebesgue measure and satisfying $G(E_Z) = I$. Then Nakamura's results are as follows.

Theorem A ([7], Theorem 1) *Let T be a pure hyponormal operator and $T = X + iY$ be the Cartesian decomposition of T. Suppose that there exists*

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