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Another general inequality for *CR*-warped products in complex space forms

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Abstract. We prove that every *CR*-warped product $N_T \times_f N_\perp$ in a complex space form $\tilde{M}^m(4c)$ of constant holomorphic sectional curvature $4c$ satisfies a general inequality: $||\sigma||^2 \geq 2p\{||\nabla(\ln f)||^2 + \Delta(\ln f)\} + 4hpc$, where $h = \dim_{\mathbf{C}} N_T$, $p = \dim_{\mathbf{R}} N_\perp$, and σ is the second fundamental form. We also completely classify *CR*-warped products in a complex space form which satisfy the equality case of this inequality.

Key words: *CR*-submanifold, *CR*-warped product, squared norm of second fundamental form, warping function, warped product, tensor product.

1. Introduction

A submanifold N of a Kähler manifold is called a *CR*-submanifold if there exists on N a differentiable holomorphic distribution $\mathcal{D}$ whose orthogonal complementary distribution $\mathcal{D}^\perp$ is a totally real distribution, i.e., $J\mathcal{D}_x^\perp \subset T_x^\perp N$ (cf. [1]). Throughout this paper we denote the complex rank of $\mathcal{D}$ by h and the real rank of $\mathcal{D}^\perp$ by p. The study of *CR*-submanifolds has been a very active field of research during the last two decades (see, for instance, [1–4, 6–9, 11, 13, 14]).

A *CR*-submanifold is called a *CR*-product if it is the direct product $N_T \times N_\perp$ of a holomorphic submanifold N_T and a totally real submanifold $N_\perp$. It was proved in [3] that a *CR*-product in a complex Euclidean space is a direct product of a holomorphic submanifold and a totally real submanifold of complex linear subspaces. It was also proved in [3] that there do not exist non-proper *CR*-products in complex hyperbolic spaces. Moreover, *CR*-products in the complex projective space CP^{h+p+hp} are obtained from the Segre imbedding in a natural way.

Let B and F be two Riemannian manifolds with Riemannian metrics g_B and g_F, respectively, and f be a positive differentiable function on B. The warped product $B \times_f F$ is the product manifold $B \times F$ equipped with the Riemannian metric $g = g_B + f^2 g_F$. The function f is called the *warping*

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