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On the univalence of an integral on a subclass of meromorphic convex univalent functions

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Abstract. A nonlinear integral operator is studied on the class of convex meromorphic functions in the exterior of the unit disk. In this paper, we improve a sufficient condition for univalency of the operator obtained earlier by the first author.

Key words: univalent, meromorphic, convex functions and integral operators.

1. Introduction and main results

Let $\mathcal{S}$ be the class of normalized functions $f(z) = z + a_2 z^2 \ldots$, analytic and univalent in the unit disk $E = \{z \in \mathbb{C} : |z| < 1\}$. In [5] an integral operator $P_\lambda[f]$ defined by

$$P_\lambda[f](z) = \int_0^z (f'(t))^\lambda \, dt$$

was shown to map $\mathcal{S}$ into itself, provided that $|\lambda| \leq (\sqrt{5}-2)/3 = 0.078\ldots$. Becker [3] established the univalence of $P_\lambda[f]$ for $|\lambda| \leq 1/6$ whereas Royster [10] gave an example implying that, unless $\lambda = 1$, for any λ outside of the disk $|\lambda| \leq 1/3$ a function $f_0 \in \mathcal{S}$ can be found such that $P_\lambda[f_0] \notin \mathcal{S}$. Pfaltzgraff [8] improved the range of λ to $|\lambda| \leq 1/4$. The question, whether the operator P_λ preserves univalency for $1/4 < |\lambda| \leq 1/3$ still remains open.

A similar problem is completely solved for the subclass $\mathcal{K} \subsetneq \mathcal{S}$ of univalent convex functions. Namely, the inclusion $P_\lambda[\mathcal{K}] \subset \mathcal{S}$ holds if and only if either $|\lambda| \leq 1/2$ or λ is real with $1/2 \leq \lambda \leq 3/2$ (see [2, 8]). More results of the similar type for other subclasses of $\mathcal{S}$ are obtained in [6, 9].

Counterparts of these problems for the case of meromorphic functions were studied by a number of authors (see [1, 2, 11]), however, the relevant constants are smaller than in the regular case. Denote by Σ be the class of function

$$f(\zeta) = \zeta + \sum_{k=0}^{\infty} \alpha_k \zeta^{-k},$$

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