Hokkaido Mathematical Journal Vol. 32 (2003) p. 291–316

Partial regularity of solutions of nonlinear quasimonotone systems

Christoph HAMBURGER

(Received November 5, 2001; Revised April 16, 2002)

Abstract. We prove partial regularity of weak solutions u of the nonlinear *strictly quasimonotone* system $\operatorname{div} A(x, u, Du) + B(x, u, Du) = 0$ under natural polynomial growth, assuming that the coefficient function $A(x, u, P)$ is Hölder continuous in (x, u) and of class C^1 in P, and that $A(x, u, P) \cdot P \geq F(x, P)$ for some continuous function F which is strictly quasiconvex at zero.

Key words: partial regularity, weak solution, nonlinear system, quasilinear system, quasimonotonicity, ellipticity, natural growth.

1. Introduction

We are interested in the regularity of the vector-valued weak solutions $u \in W^{1,2}(\Omega, \mathbf{R}^N)$ of the nonlinear system

$$\operatorname{div} A(x, u, Du) + B(x, u, Du) = 0, \tag{1}$$

or, in components,

$$\sum_{\alpha=1}^{n} D_\alpha \left(A_i^\alpha (x, u, Du)\right) + B_i (x, u, Du) = 0 \quad \text{for} \quad i = 1, \ldots, N.$$

Here Ω is a bounded open subset of $\mathbf{R}^n$, $n \geq 2$, $N \geq 1$, and $Du(x) \in \mathbf{R}^{N \times n}$ denotes the gradient of u at a.e. point $x \in \Omega$. The coefficient functions A and B are defined on the set

$$\mathfrak{Z} = \overline{\Omega} \times \mathbf{R}^N \times \mathbf{R}^{N \times n}$$

with values in $\mathbf{R}^{N \times n}$ and $\mathbf{R}^N$ respectively.

Definition 1 We say that $u \in W^{1,2}(\Omega, \mathbf{R}^N)$ is a *weak solution* of the system (1) if $A(x, u, Du)$ and $B(x, u, Du)$ are locally integrable and

$$\int_\Omega A(x, u, Du) \cdot D\varphi \, dx = \int_\Omega B(x, u, Du) \cdot \varphi \, dx \tag{2}$$

2000 Mathematics Subject Classification : 35J60, 35B65, 35D10.