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On construction of continuous functions with cusp singularities

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Abstract. In this paper, we study various constructions of continuous functions on $\mathbf{R}$ which have the prescribed cusp singularities at each point. As applications, we get some generalizations of the results given in our previous paper [7], which discuss the cusp singularities of the classical Weierstrass functions and Takagi function.

Key words: wavelets, scaling exponents, singularities, Weierstrass functions, spline functions, Takagi function.

1. Introduction

Let s be a positive number, which is not an integer and let x_0 be a point in $\mathbf{R}^n$. Then a function f on $\mathbf{R}^n$ belongs to the pointwise Hölder space $C^s(x_0)$, if there exists a polynomial P of degree less than s such that

$$|f(x)-P(x-x_0)|\leq C|x-x_0|^s$$

in a neighborhood of x_0. The pointwise Hölder exponent of a function f at a point x_0 in $\mathbf{R}^n$ is defined as

$$H(f,x_0)=\sup\{s>0; f\in C^s(x_0)\}.$$

If a continuous function f does not belong to $C^s(x_0)$ for every $s>0$, then $H(f,x_0)=0$.

However the pointwise Hölder exponent of a function f at a point x_0 in $\mathbf{R}^n$ is not stable under the pseudo-differential operators. Similarly it does not fully characterize the oscillatory behavior on a neighborhood of x_0. This implies that $f\in C^s(x_0)$ cannot be characterized by size estimates on the wavelet coefficients of f.

Here let us recall the definition of the weak scaling exponent characterizing the local oscillatory behavior.

$\mathcal{S}_0(\mathbf{R}^n)$ denotes the closed subspace of the Schwartz class $\mathcal{S}(\mathbf{R}^n)$ such

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