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Pseudo-eigenvalues of W-operators on Hilbert modular forms

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Abstract. In this paper, we study pseudo-eigenvalues of W-operators on Hilbert modular forms. In particular, we show that they are roots of unity under a certain condition.

Key words: Hilbert modular forms.

1. Introduction

Let F be a totally real number field. For a positive integer k, an integral ideal $\mathfrak{N}$ of F and a (Hecke) character ψ, we consider the space $S_k^0(\mathfrak{N},\psi)$ of new forms of weight k, level $\mathfrak{N}$ and character ψ (see §2 for the definition). We have operators $\eta_{\mathfrak{p}}$ (W-operator) on $S_k^0(\mathfrak{N},\psi)$ for each prime $\mathfrak{p}$ dividing $\mathfrak{N}$ (see §2 for the definition and details). For a primitive form $\mathbf{f}$ of $S_k^0(\mathfrak{N},\psi)$, we can write $\mathbf{f}|\eta_{\mathfrak{p}}=c\mathbf{g}$ with the corresponding primitive form $\mathbf{g}$ and call $c=c_{\mathfrak{p}}$ the pseudo-eigenvalue of $\eta_{\mathfrak{p}}$ associated to $\mathbf{f}$. In the case that the $\mathfrak{p}$-th Fourier coefficient of $\mathbf{f}$ does not vanish, the pseudo-eigenvalue of $\eta_{\mathfrak{p}}$ associated to $\mathbf{f}$ is expressed by the $\mathfrak{p}$-th Fourier coefficient of $\mathbf{f}$ and the local Gauss sum (Corollary 2.7). In the case that the $\mathfrak{p}$-th Fourier coefficient of $\mathbf{f}$ vanishes, we have the following theorem in the simplest case.

Theorem 1.1 *Let $\mathfrak{p}$ be a prime ideal of F, $\mathbf{f}$ a primitive form of $S_k^0(\mathfrak{p}^e,\psi)$ and $\mathfrak{p}^n$ the conductor of ψ. If $3n\leq e$, the pseudo-eigenvalue A associated to $\mathbf{f}$ satisfies $A^{2\alpha}=(\psi^\alpha)^*(\mathfrak{p}^e)$, where α is the order of $\psi_{\mathfrak{p}}$ as the character of $(\mathfrak{o}_F/\mathfrak{p}^n)^\times$ and $(\psi^\alpha)^*$ the ideal character associated with ψ^α. Moreover if ψ is of finite order, the pseudo-eigenvalue is a root of unity.*

In the next section, we will introduce necessary notations for adelic Hilbert modular forms. In §3, we study twisted forms, since primitive forms whose Fourier coefficients of level parts vanish may be twisted forms. In the last section, we prove above theorem in more general case (Theorem 4.3). We note that we consider a unitary Hecke character (possibly of infinite

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