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Subclasses of certain analytic functions

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Abstract. Let $\mathcal{A}$ be the class of functins $f(z)$ which are analytic in the open unit disc $\mathbb{E}$ with $f(0)=0$ and $f'(0)=1$. Two subclasses of $\mathcal{A}$ with some inequalities are defined. The object of the present paper is to consider some properties for functions $f(z)$ belonging to these classes.

Key words: analytic function, univalent function, starlike function, subordination.

1. Introduction

Let $\mathcal{A}$ denote the class of functions $f(z)$ of the form

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n$$

which are analytic in the open unit disc $\mathbb{E}=\{z\in\mathbb{C}:|z|<1\}$. We denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions $f(z)$ which are univalent in $\mathbb{E}$. A function $f(z)\in\mathcal{A}$ is called starlike in $|z|<r$ $(0<r\leqq 1)$ if it satisfies

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\}>0 \qquad (|z|<r).$$

For a function $f(z)\in\mathcal{A}$, we say that $f(z)$ is in the class $\mathcal{H}(\lambda,\mu)$ if and only if it satisfies the conditions $\frac{f(z)}{z}\neq 0$ $(z\in\mathbb{E})$ and

$$\left|\frac{z^2f'(z)}{f(z)^2}-\lambda z^2\left(\frac{z}{f(z)}\right)''-1\right|<\mu \qquad (z\in\mathbb{E}), \tag{1}$$

where λ is a complex number with $\operatorname{Re}(\lambda)\geqq 0$ and μ is a positive real number. Also we define the class $\mathcal{H}_0(\lambda,\mu)$ by

$$\mathcal{H}_0(\lambda,\mu)=\{f(z)\in\mathcal{H}(\lambda,\mu):f''(0)=0\}.$$

Nunokawa, Obradović and Owa [2] proved that if $f(z)\in\mathcal{A}$ with

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