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Generalized shadows of codes over rings

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Abstract. We describe different ways of defining shadows for self-dual codes over rings, giving special attention to the rings of order 4. We determine their respective weight enumerators and give the corresponding shadow sum constructions. We also give a connection between the shadow of a code and its construction via the Chinese Remainder Theorem.

Key words: self-dual codes, shadows, codes over rings.

1. Introduction

Self-dual codes over rings have become an important object of study. They are interesting as objects themselves, however they have added importance because of their relationship to real and complex unimodular lattices and their corresponding theta series, which are then used to construct modular forms.

Self-dual codes over $\mathbf{Z}_{2k}$ were introduced in [2] and have been studied extensively elsewhere, see [13]. Self-dual codes over $\mathbf{Z}_4$ have been widely investigated (see [11] and the references given therein). Self-dual code over $\mathbf{F}_2+u\mathbf{F}_2$ were introduced in [1] and [5]. Three classes namely Type I, Type II and Type IV codes were introduced in [2] and [6].

We shall describe the theory of shadows applied to codes over the rings $\mathbf{Z}_k$ and $\mathbf{F}_2+u\mathbf{F}_2$. Moreover we shall show how this theory can be applied in different manners.

Shadows for binary codes were introduced in [4]. The definition was generalized for codes over $\mathbf{Z}_4$ in [7], for codes over $\mathbf{Z}_{2k}$ in [2], and for codes over $\mathbf{F}_2+u\mathbf{F}_2$ in [5]. In [9], a detailed study of these shadows and the corresponding shadows for lattices is given.

There are two primary purposes for shadows:

1) Eliminate a putative code by examining the weight enumerator (Hamming, Symmetric, or Complete) of the shadow by finding a coefficient

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