Nonlinear Schrödinger equations with Stark potential

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Abstract. We study the nonlinear Schrödinger equations with a linear potential. A change of variables makes it possible to deduce results concerning finite time blow up and scattering theory from the case with no potential.

Key words: nonlinear Schrödinger equations, Stark effect, Avron-Herbst formula, pseudo-conformal conservation law, finite time blow up.

1. Introduction

In this note, we consider the nonlinear Schrödinger equation with Stark effect,

\[
\begin{aligned}
    i\varepsilon \partial_t u + \frac{1}{2} \varepsilon^2 \Delta u &= V(x)u + \lambda |u|^{2\sigma} u, \\
    u|_{t=0} &= u_0,
\end{aligned}
\]  

(1.1)

where \( x \in \mathbb{R}^n \), and the potential \( V \) is linear,

\[
    V(x) = E \cdot x; \ E = (E_1, \ldots, E_n) \in \mathbb{R}^n \setminus \{0\}.
\]  

(1.2)

We assume that \( \varepsilon \in ]0, 1] \), \( \lambda \in \mathbb{R} \), \( \sigma > 0 \), and \( \sigma < 2/(n-2) \) if \( n \geq 3 \).

The vector \( E \) may represent a constant electric field (see e.g. [6]), or gravity (see e.g. [25]). We introduce the factor \( \varepsilon \) to treat both the quantum case where \( \varepsilon = \hbar \) (see e.g. [25]), and the case \( \varepsilon = 1 \), where the nonlinear Schrödinger equation may appear as an envelope equation (see e.g. [24], [8], [13]).

We compare solutions of (1.1) with solutions to the nonlinear Schrödinger equation,

\[
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\]

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