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On approximation of 2π-periodic functions in Hölder spaces

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Abstract. This note is connected with results given in papers [2-5]. We give two approximation theorems for 2π-periodic functions belonging to generalized Hölder spaces. We present also applications of these theorems.

Key words: Hölder space, approximation theorem, de la Vallée Poussin integral, Abel means.

1. Preliminaries

1.1. Let $L_{2\pi}^{p}$, $1 \leq p \leq \infty$, be the space of 2π-periodic real-valued functions, Lebesgue integrable with p-th power on $[-\pi, \pi]$ if $1 \leq p < \infty$ and continuous on $R := (-\infty, +\infty)$ if $p = \infty$. Let the norm of f in $L_{2\pi}^{p}$ be defined by

$$\|f\|_p \equiv \|f(\,\cdot\,)\|_p := \begin{cases} \left(\displaystyle\int_{-\pi}^{\pi} |f(x)|^p \, dx\right)^{1/p} & \text{if} \quad 1 \leq p < \infty, \\ \displaystyle\max_{|x| \leq \pi} |f(x)| & \text{if} \quad p = \infty. \end{cases} \tag{1}$$

For $f \in L_{2\pi}^{p}$, we define as usual ([7]) the modulus of continuity $\omega_1(f, p; \cdot)$ and the modulus of smoothness $\omega_k(f, p; \cdot)$ of the order $2 \leq k \in N := \{1, 2, \ldots\}$ by the formula

$$\omega_k(f, p; t) := \sup_{|h| \leq t} \|\Delta_h^k f(\,\cdot\,)\|_p, \quad t \geq 0, \tag{2}$$

where

$$\begin{aligned} \Delta_h^1 f(x) &:= f(x+h) - f(x), \\ \Delta_h^k f(x) &:= \Delta_h^1\big(\Delta_h^{k-1} f(x)\big) \quad \text{if} \quad k \geq 2. \end{aligned} \tag{3}$$

Hence, we have

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