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The global existence theorem for quasi-linear wave equations with multiple speeds

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Abstract. The Cauchy problem is studied for systems of quasi-linear wave equations with multiple speeds. We pursue the extension of the excellent method of Klainerman and Sideris to its limit, and a unified proof is given to previous results of Agemi-Yokoyama, Hoshiga-Kubo, Kovalyov, and Yokoyama.

Key words: global existence, quasi-linear wave equations, non-resonance.

1. Introduction

The well-known commuting vector fields method of John and Klainerman has brought remarkable progress in the study of large-time existence of small amplitude solutions to quasi-linear wave equations. Its feature lies in the fact that only the energy integral argument together with the Klainerman inequality [20], [22] suffices to prove almost global existence for perturbed classical wave equations if quasi-linear perturbation terms are quadratic in three space dimensions or cubic in two space dimensions. The Klainerman inequality contains the boost operators which are members of the generators of Lorentz rotations in the Minkowski space. In the analysis of elastic equations as well as systems of multiple-speed wave equations, a lack of the Lorentz invariance used to be compensated for by direct estimates of the fundamental solution to obtain some L^{∞}-decay estimates in the late 1980's [13], [24]. In the middle of 1990's Klainerman and Sideris threw a new light on the difficulty of lack of the Lorentz invariance. They have successfully overcome it, and proved almost global existence for quadratic quasi-linear wave equations in three space dimensions without relying on direct estimates of the fundamental solution [23]. In their analysis Klainerman and Sideris made use of only the invariance of the D'Alembertian operator under Euclid rotations, space-time scaling in addition to space-time translations. Though the spatial divergence form is assumed in the

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