Maximum rank webs are not necessarily almost Grassmannizable

Vladislav V. GOLDBERG

(Received September 27, 2002; Revised April 16, 2003)

Abstract. We present an example of a 6-web W(6, 3, 2) of codimension two and of maximum rank on a six-dimensional manifold which is not almost Grassmannizable.

Key words: web, maximum rank, almost Grassmannizable web.

0. Introduction

In 1984, during the Problem Session in the meeting on web geometry at the Mathematisches Forschungsinstitut Oberwolfach, Goldberg posed the following problem (see [8]):

6. Every d-web W(d, n, r) of maximum r-rank is almost Grassmannizable. Is it true or wrong?"

Little in his paper [7] related the Chern and Griffiths approach in studying the rank problem and Grassmannization and algebraization problems for webs based on the presence of abelian equations on the web with the Akivis and Goldberg approach based on the notion of the almost Grassmannizable web. He wrote in the introduction: "The major purpose of this paper is to relate these two approaches by showing that, in rough terms, if d is large enough (relative to r, n) so that $\pi(d, n, r) > 1$, then every maximum rank web W(d, n, r) is almost-Grassmannizable. This answers (part of) a question posed by Goldberg at the 1984 Oberwolfach Conference on web geometry."

Little proved in [7] that if r > d(n-1)+2, then every maximum r-rank web W(d, n, r) is almost Grassmannizable.

In [7] Little also considered maximum r-rank webs W(d, n, r) with d = r(n-1) + 2. In particular, he proved that "the maximum 2-rank webs W(6, 3, 2) are also almost Grassmannizable" (see his Example 1). It appeared that this last Little's result was wrong.