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Germs of Engel structures along 3-manifolds

(Dedicated to Professor Hajime Sato on his sixtieth birthday)

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Abstract. Engel structures along an embedded 3-manifold in a 4-manifold are studied in this article. It is shown that, among the Engel structures having the same oriented even-contact structure around the given embedded closed oriented 3-manifold as derived distributions with the induced orientations, the germ of Engel structure along the 3-manifold is determined by the singular line field on the 3-manifold traced by the Engel structure.

Key words: Engel structures, even-contact structures.

1. Introduction

A tangent distribution of rank 2, maximally non-integrable, on a 4-dimensional manifold is called an *Engel structure*. Engel structures have a special property: all Engel structures are locally equivalent (see [BCG3]). Such a phenomenon occurs only for line fields, contact structures, even-contact structures, and Engel structures among generic regular distributions (see [VG], [Mon1]). Therefore studying global properties of Engel structures is important. We study Engel structures along an embedded 3-manifold, and investigate the conditions under which germs of Engel structures are determined up to isotopy.

Although an Engel structure has similar properties to a contact structure, they have some differences. A contact structure is also defined as a maximally non-integrable hyperplane field on an odd-dimensional manifold. Contact structures on a closed manifold have global stability for deformations: any two contact structures on a closed manifold which are homotopic among contact structures are isotopic (Gray's stability Theorem [Gr]). However, it is known that the corresponding statement for Engel structures does not hold (see [Ge1]). A homotopy of Engel structures is not always represented by an isotopy. An obstruction for the isotopy is considered as follows. An Engel structure D defines a line field $L(D^2) \subset$

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