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Operators having commutants endowed with cyclicity-preserving quasiaffinities

(Dedicated to the memory of Katsutoshi Takahashi)

László KÉRCHY*

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Abstract. It is shown that there are commutant-cyclic vectors in the ranges of the quasiaffinities belonging to the commutant of any isometry or any quasinormal operator with a dominating unilateral shift part. This property ensures that the commutant-multiplicity is constant in the quasisimilarity orbits of these operators.

Key words: commutant, cyclic vector, multiplicity, quasiaffinity, quasisimilarity, isometry, quasinormal operator, outer function.

1. Introduction

Let $\mathcal{H}$ be a (nonzero, separable, complex) Hilbert space, and let $\mathcal{L}(\mathcal{H})$ denote the C^*-algebra of all (bounded, linear) operators acting on $\mathcal{H}$. Given a subalgebra $\mathcal{A}$ of $\mathcal{L}(\mathcal{H})$, containing the identity operator I, a nonempty vector set $\mathcal{G} \subset \mathcal{H}$ is called *cyclic* for $\mathcal{A}$, if the vectors $\mathcal{A}\mathcal{G} = \{Ag\colon A \in \mathcal{A},\ g \in \mathcal{G}\}$ span the whole space: $\vee\mathcal{A}\mathcal{G} = \mathcal{H}$. The minimum of the cardinalities $|\mathcal{G}|$ of the sets $\mathcal{G}$, cyclic for $\mathcal{A}$, is called the *multiplicity* of $\mathcal{A}$, and is denoted by $\mu(\mathcal{A})$. With an operator $T \in \mathcal{L}(\mathcal{H})$ two algebras can be naturally associated: the algebra $\mathcal{A}_T := \{p(T)\colon p(\lambda) \text{ is a polynomial}\}$ generated by T, and the commutant $\{T\}' := \{C \in \mathcal{L}(\mathcal{H})\colon CT = TC\}$ of T. The multiplicity of $\mathcal{A}_T$ is called the *multiplicity of the operator* T, and is denoted by $\mu(T) := \mu(\mathcal{A}_T)$. The multiplicity of $\{T\}'$ is called the *commutant-multiplicity of* T, and is denoted by $\mu'(T) := \mu(\{T\}')$. A quick inspection in well-known classes of operators convince the reader that while the multiplicity $\mu(T)$ shows great variety, the commutant is usually cyclic, that is $\mu'(T) = 1$. It was W.R. Wogen who showed in [W] that the commutant-multiplicity $\mu'(T)$ can be also arbitrary. Even more, it turned out that, for any cardinal number $1 \leq n \leq \aleph_0$, the set $\mathcal{C}_n(\mathcal{H}) = \{T \in \mathcal{L}(\mathcal{H})\colon \mu'(T) = n\}$ is norm-dense in the operator space $\mathcal{L}(\mathcal{H})$, provided $\dim \mathcal{H} = \aleph_0$ (see [AFHV, Theorem 11.19]).

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