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Local isometric imbeddings of $P^2(\boldsymbol{H})$ and $P^2(\boldsymbol{Cay})$

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Abstract. We investigate local isometric imbeddings of the quaternion projective plane $P^2(\boldsymbol{H})$ and the Cayley projective plane $P^2(\boldsymbol{Cay})$ into the Euclidean spaces. We prove a non-existence theorem of local isometric imbeddings (see Theorem 2), by which we can conclude that the isometric imbeddings given in Kobayashi [8] are the least dimensional isometric imbeddings of $P^2(\boldsymbol{H})$ and $P^2(\boldsymbol{Cay})$.

Key words: Pseudo-nullity, isometric imbedding, projective plane.

1. Introduction

In this paper we investigate local isometric imbeddings of the quaternion projective plane $P^2(\boldsymbol{H})$ and the Cayley projective plane $P^2(\boldsymbol{Cay})$ into the Euclidean spaces.

In [5], we determined the pseudo-nullity $p(G/K)$ for each compact rank one symmetric space G/K. (For the definition of the pseudo-nullity, see [5].) Utilizing $p(G/K)$, we have obtained the following result concerning the non-existence of isometric imbeddings of the complex projective spaces $P^n(\boldsymbol{C})\,(n \geq 2)$, the quaternion projective spaces $P^n(\boldsymbol{H})\,(n \geq 2)$ and the Cayley projective plane $P^2(\boldsymbol{Cay})$ (see Theorem 5.6 of [5]).

Theorem 1 *Let G/K be one of the complex projective space $P^n(\boldsymbol{C})$ $(n \geq 2)$, the quaternion projective space $P^n(\boldsymbol{H})$ $(n \geq 2)$ and the Cayley projective plane $P^2(\boldsymbol{Cay})$. Define an integer $q(G/K)$ by setting $q(G/K) = 2\dim G/K - p(G/K)$, i.e.,*

$$q(G/K) = \begin{cases} \min\{4n-2,\, 3n+1\}, & \text{if} \quad G/K = P^n(\boldsymbol{C}) \ (n \geq 2), \\ \min\{8n-3,\, 7n+1\}, & \text{if} \quad G/K = P^n(\boldsymbol{H}) \ (n \geq 2), \\ 25, & \text{if} \quad G/K = P^2(\boldsymbol{Cay}). \end{cases}$$

Then, any open set of G/K cannot be isometrically imbedded into the Euclidean space $\boldsymbol{R}^Q$ with $Q \leq q(G/K) - 1$.

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