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Non-closed minimal hypersurfaces of $\mathbb{S}^4(1)$ with identically zero Gauß-Kronecker curvature

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Abstract. We give a partial local description of minimal hypersurfaces M^3 with identically zero Gauß-Kronecker curvature function in the unit 4-sphere $\mathbb{S}^4(1)$, without assumption on the compactness of M^3.

Key words: minimal hypersurfaces in spheres, isoparametric hypersurfaces, identically zero Gauß-Kronecker curvature, nowhere zero second fundamental form.

1. Introduction

Let $x\colon M^3 \longrightarrow \mathbb{S}^4(1) \subset \mathbb{R}^5$ be a hypersurface immersion of a connected and orientable 3-dimensional manifold M^3 of class C^∞ into $\mathbb{S}^4(1) \subset \mathbb{R}^5$. Let λ_1, λ_2 and λ_3 be the three principal curvature functions. The normalized elementary symmetric curvature functions of the immersion x are given by:

$$H := \frac{1}{3}(\lambda_1 + \lambda_2 + \lambda_3),$$

$$H_2 := \frac{1}{3}(\lambda_1\lambda_2 + \lambda_1\lambda_3 + \lambda_2\lambda_3),$$

$$K := \lambda_1\lambda_2\lambda_3.$$

S. Almeida and F. Brito [1] suggested to classify closed hypersurface immersions for which two of the three functions H, H_2, K are constant. The paper [3] gives a survey of results on closed hypersurfaces in $\mathbb{S}^4(1)$ with two constant curvature functions.

Particularly, the paper [2] investigated closed minimal hypersurfaces with constant Gauß-Kronecker curvature function, corresponding to $H \equiv 0$ and $K \equiv$ const. There it is proved that closed minimal hypersurfaces with constant Gauß-Kronecker curvature $K \neq 0$ are isoparametric, therefore closed minimal hypersurfaces with constant Gauß-Kronecker curvature

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