*Hokkaido Mathematical Journal Vol. 33 (2004) p. 127–151*

# Dominated semigroups of operators and evolution processes

Brian JEFFERIES and Susumu OKADA

(Received April 3, 2002)

**Abstract.** A semigroup $S$ acting on a Banach lattice $E$ is said to be dominated if there exists a positive semigroup $T$ such that $|S(t)x| \leq T(t)|x|$ for all $x \in E$ and $t > 0$. It is shown that a semigroup on $L^p$ is dominated if and only if it is associated with a family of operator valued measures.

*Key words*: dominated semigroup, evolution equation, modulus, regular operator, variation.

## Introduction

The problem of determining when a semigroup $S$ of operators acting on a Dedekind complete Banach lattice is dominated by a semigroup $T$ of positive operators is an old one. Because $|S(t)x| \leq T(t)|x|$ for each element $x$, it follows that $S(t)$ must be a regular operator for each $t \geq 0$. A related question is when does the smallest such semigroup $|S|$ exist — the *modulus semigroup* of $S$. Although a $C_0$-*contraction* semigroup on $L^1$ is dominated by a positive semigroup (see [12], [14]), C. Kipnis [12, pp. 374–376] gives an example of a $C_0$-semigroup on $\ell^1$ which is not dominated by any positive semigroup. Other examples are provided by the semigroups $T_z$ mentioned below with $\Im z > 0$ and $\Re z \neq 0$. A sufficient condition that a semigroup $S$ on an $L^p$-space be dominated by a positive semigroup is provided by I. Becker and G. Greiner [3, Proposition 2.3]: if there exists a real number $\omega$ such that $\| \, |S(t)| \, \|_{\mathcal{L}(L^p)} \leq e^{\omega t}$ for all $t \geq 0$ (that is, $S$ is *quasicontractive with respect to the regular norm*), then $S$ is dominated and if, in addition, $S$ is a $C_0$-semigroup, then the modulus semigroup $|S|$ is also a $C_0$-semigroup.

Regular operators $R$ acting on $L^p$-spaces were characterised in [10] in terms of an operator bound of the form

---

*2000 Mathematics Subject Classification* : Primary 47D06, 28B15; Secondary 47B65.

The research of the first author was supported by the Australian Research Council.