Michigan Math. J. 66 (2017), 533–547

Hypercommutative Algebras and Cyclic Cohomology

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Abstract. We introduce a chain model for the Deligne–Mumford operad formed by homotopically trivializing the circle in a chain model for the framed little disks. We then show that under degeneration of the Hochschild to cyclic cohomology spectral sequence, a known action of the framed little disks on Hochschild cochains lifts to an action of this new chain model. We thus establish homotopy hypercommutative algebra structures on both Hochschild and cyclic cochain complexes, and we interpret the gravity brackets on cyclic cohomology as obstructions to degeneration of this spectral sequence. Our results are given in the language of deformation complexes of cyclic operads.

Introduction

A differential graded Batalin–Vilkovisky (BV) algebra enhanced with a homotopy trivialization of the Δ-operator is equivalent to a hypercommutative (HyCom) algebra [DCV13; KMS13; DC14]. This relationship may be described in the language of operads, where BV and HyCom algebras are represented respectively by the homology operads of genus 0 moduli spaces of surfaces with boundary [Get94a] and by the Deligne–Mumford compactification of the moduli space of surfaces with punctures [Get95]. In practice, BV algebras often arise as the homology or cohomology of a geometric, topological, or algebraic object, and the chain level structure can only be expected to be BV up to homotopy. For example, this is the case when studying Hochschild cochain operations via the cyclic Deligne conjecture [Kau08]. More generally, this is the case when considering the deformation complex of a cyclic operad $\mathcal{O}$ with A_∞ multiplication μ. We denote such a deformation complex $CH^*(\mathcal{O}, \mu)$.

On the other hand, the results of [War16] show that the complex of cyclic invariants associated with such data carries a compatible structure of an algebra over a model of the *open* moduli space of punctured Riemann spheres. This complex of invariants is a generalization of Connes' C^*_λ-complex and will be denoted $C^*_\lambda(\mathcal{O}, \mu)$. Its cohomology $HC^*(\mathcal{O}, \mu)$ generalizes the notion of the cyclic cohomology of a cyclic k-module. It is natural to ask for conditions under which this action of the open moduli space lifts to an action of an operad of chains on the Deligne–Mumford compactification.

In a BV algebra, the Δ-operator corresponds to an action of the circle at a boundary component. In the homotopy theory of S^1-spaces, trivialization of the circle action corresponds to degeneration of the Hochschild to cyclic

Received February 18, 2016. Revision received September 29, 2016.