Michigan Math. J. 66 (2017), 309–346

Rational Singularities, ω-Multiplier Ideals, and Cores of Ideals

Kohsuke Shibata

Abstract. We define the ω-multiplier ideals on a normal variety. The main goal of this paper is to introduce an ω-multiplier ideal and prove its properties. We give characterizations of two-dimensional rational singularities by means of ω-multiplier ideals and cores of ideals.

1. Introduction

In this paper, we always assume that a ring is a domain essentially of finite type over $\mathbb{C}$ and a variety is an irreducible reduced separated scheme of finite type over $\mathbb{C}$.

Rees and Sally [27] introduced the cores of ideals. Okuma, Watanabe, and Yoshida [26] characterized a two-dimensional local ring with rational singularity via cores of ideals. However, in the higher-dimensional case, we have a counterexample to such a characterization. We give another characterization of a local ring with rational singularity of arbitrary dimension via cores of ideals. We, namely, will prove the following:

Theorem 1.1. *Let $(A, \mathfrak{m})$ be an n-dimensional Cohen–Macaulay local ring with an isolated singularity. Then A is a rational singularity if and only if $\overline{I^n} \subset \operatorname{core}(I)$ for any $\mathfrak{m}$-primary ideal I.*

By this theorem, we show that a Cohen–Macaulay local ring with an isolated singularity has a rational singularity if the Briançon–Skoda theorem holds for the ring. Lipman and Teissier [23] showed that the Briançon–Skoda theorem holds for a local ring with rational singularities. Therefore a Cohen–Macaulay local ring with an isolated singularity has a rational singularity if and only if the Briançon–Skoda theorem holds for the ring.

The multiplier ideals are fundamental tools in birational geometry. In this paper, we introduce a new notion of an "ω-multiplier ideal", which has similar properties and works in a slightly different way than a multiplier ideal. The main goal of this paper is to prove the properties of ω-multiplier ideals and show some applications.

For the definition of the multiplier ideals, we use the discrepancies. In order for the discrepancy to be well defined, we need to assume that the variety is normal

Received January 8, 2016. Revision received January 27, 2017.
The author was partially supported by the Program for Leading Graduate Schools, MEXT, Japan and JSPS KAKENHI Grant Number 15-J09158.