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A Note on Cabled Slice Knots and Reducible Surgeries

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ABSTRACT. We consider the question of when a slice knot admits a reducible Dehn surgery. By analyzing the correction terms associated to such a surgery we show that slice knots cannot admit surgeries with more than two summands. We also give a necessary Heegaard Floer theoretic condition for a positive cable of a knot to be slice.

1. Introduction

Dehn surgery is one of the simplest and most important operations in three-manifold topology, and understanding which three-manifolds can result from Dehn surgery on a knot in S^3 has been one of the major goals of modern low-dimensional topology. We refer the reader to [6] for an introduction and comprehensive overview of the history and scope of this endeavor.

Perhaps the most basic question related to this goal is when the result of Dehn surgery can be a reducible manifold (a manifold containing an essential two-sphere). Moser's classification of Dehn surgeries on torus knots [20] gave the first nontrivial examples of this phenomenon:

$$S^3_{pq}(T_{p,q}) \cong L(p,q)\#L(q,p).$$

Of course, the fact that $S^3_0(U) \cong S^1 \times S^2$ can be thought of as a degenerate case of this. Gabai [3] showed that no other knot in S^3 admits a surgery to $S^1 \times S^2$, so we may assume that any nontrivial reducible surgery decomposes as a connected sum.

Other interesting examples of reducible surgeries come from considering cabled knots [7]. Let $J_{p,q}$ denote the (p,q)-cable of J for some pair $p, q \in \mathbb{Z}$ with $p \geq 2$. (Throughout, p is the longitudinal winding number.) Then we have

$$S^3_{pq}(J_{p,q}) \cong L(p,q)\#S^3_{q/p}(J).$$

Note that this generalizes the case of torus knots, which can be thought of as cables of the unknot. Cabled knots represent the only known examples of knots admitting reducible surgeries, and we have the following conjecture of Gonzales Acuña and Short.

CABLING CONJECTURE ([4]). *If K is a nontrivial knot in S^3 and $S^3_r(K)$ is reducible, then $K = J_{p,q}$ for some knot J, and $r = pq$.*

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