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On Generic Vanishing for Pluricanonical Bundles

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Abstract. We study cohomology support loci and higher direct images of (log) pluricanonical bundles of smooth projective varieties or log canonical pairs. We prove that the zeroth cohomology support loci of log pluricanonical bundles are finite unions of torsion translates of subtori, and we give a generalization of the generic vanishing theorem to log canonical pairs. We also construct an example of morphism from a smooth projective variety to an Abelian variety such that a higher direct image of a pluricanonical bundle to the Abelian variety is not a GV-sheaf.

1. Introduction

Throughout this paper, we always assume that all varieties are defined over the complex number field. Let X be a smooth projective variety, and Δ be a simple normal crossing divisor on X. In this paper, we prove some results of generic vanishing theory for $K_X + \Delta$ or, more generally, for $m(K_X + \Delta)$ for any positive integer m.

In Section 3, we prove some results about the structure of cohomology support loci for a log canonical pair (X, Δ) (for the definition of cohomology support loci, see Section 2.1). Originally, Simpson [Sim] proved the following theorem.

Theorem 1.1 (Simpson). *Let X be a smooth projective variety. Then the cohomology support locus*

$$S^i_j(K_X) = \{\xi \in \mathrm{Pic}^0(X) \mid h^i(X, \mathcal{O}_X(K_X) \otimes \xi) \geq j\}$$

is a finite union of torsion translates of Abelian subvarieties of $\mathrm{Pic}^0(X)$ *for any* $i \geq 0$ *and* $j \geq 1$.

In [ClHa], a generalization of Theorem 1.1 for Kawamata log terminal pairs is discussed (see [ClHa, Thm. 8.3]). We generalize Theorem 1.1 for log canonical pairs.

Theorem 1.2 (= Theorem 3.5). *Let X be a smooth projective variety, Δ be a boundary $\mathbb{Q}$-divisor on X, that is, a $\mathbb{Q}$-divisor whose coefficients are in* $[0, 1]$, *with simple normal crossing support, $f : X \to A$ be a morphism to an Abelian variety, and D be a Cartier divisor on X such that $D \sim_{\mathbb{Q}} K_X + \Delta$. Then the cohomology support locus*

$$S^i_j(D, f) = \{\xi \in \mathrm{Pic}^0(A) \mid h^i(X, \mathcal{O}_X(D) \otimes f^*\xi) \geq j\}$$

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