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The Parabolic Infinite-Laplace Equation in Carnot Groups

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Abstract. By employing a Carnot parabolic maximum principle we show the existence and uniqueness of viscosity solutions to a class of equations modeled on the parabolic infinite Laplace equation in Carnot groups. We show the stability of solutions within the class and examine the limit as t goes to infinity.

1. Motivation

In Carnot groups, the following theorem has been established.

Theorem 1.1 [3; 14; 5]. *Let Ω be a bounded domain in a Carnot group, and let $v : \partial\Omega \to \mathbb{R}$ be a continuous function. Then the Dirichlet problem*

$$\begin{cases} \Delta_\infty u = 0 & \text{in } \Omega, \\ u = v & \text{on } \partial\Omega \end{cases}$$

has a unique viscosity solution u_∞.

Our goal is to prove a parabolic version of Theorem 1.1 for a class of equations (defined in the next section), namely:

Conjecture 1.2. Let Ω be a bounded domain in a Carnot group, and let $T > 0$. Let $\psi \in C(\overline{\Omega})$ and $g \in C(\Omega \times [0, T))$ Then the Cauchy–Dirichlet problem

$$\begin{cases} u_t - \Delta_\infty^h u = 0 & \text{in } \Omega \times (0, T), \\ u(x, 0) = \psi(x) & \text{on } \overline{\Omega}, \\ u(x, t) = g(x, t) & \text{on } \partial\Omega \times (0, T) \end{cases} \tag{1.1}$$

has a unique viscosity solution u.

In Sections 2 and 3, we review key properties of Carnot groups and parabolic viscosity solutions. In Section 4, we prove the uniqueness, and Section 5 covers the existence.

2. Calculus on Carnot Groups

We begin by denoting an arbitrary Carnot group in $\mathbb{R}^N$ by G and its corresponding Lie algebra by g. Recall that g is nilpotent and stratified, resulting in the

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