## Strong Regular Embeddings of Deligne–Mumford Stacks and Hypertoric Geometry

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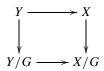
ABSTRACT. We introduce the notion of *strong regular embeddings* of Deligne–Mumford stacks. These morphisms naturally arise in the related contexts of generalized Euler sequences and hypertoric geometry.

## 1. Introduction

Let *G* be a finite group acting on affine schemes X = Spec A and Y = Spec B, and let  $f: Y \to X$  be a *G*-equivariant morphism. Since *f* is *G*-equivariant, there is an induced map of invariant subrings  $A^G \to B^G$  corresponding to a morphism of quotients  $g: Y/G \to X/G$ . Certain algebro-geometric properties of the morphism *f* (typically related to finiteness) are automatically preserved by the morphism *g*. For example, if *f* is finite, then the induced morphism of quotients  $g: Y/G \to X/G$  is also finite. Likewise, if |G| is a unit in Spec *B* and  $f: Y \to X$ is a closed embedding, then  $g: Y/G \to X/G$  is as well.

On the other hand, many properties of the morphism f will not descend. If f is flat or smooth, the induced morphism of quotients need not be. Instead, we can impose additional conditions on the actions of G on X and Y to ensure that a property of morphisms of schemes does descend to the quotient. Two obvious conditions that suffice are that G act freely or that G act trivially on both spaces.

Note, however, that these conditions are not necessary. For example, if  $Y = X \times Z$  and *G* acts trivially on *Z*, then  $Y/G = X/G \times Z$ , so the flat projection  $Y \to X$  descends to a flat projection  $Y/G \to X/G$ , and the diagram



is Cartesian.

This is an example of a *stabilizer preserving morphism*, meaning that for every point  $y \in Y$ , the map of stabilizers  $\operatorname{Stab}_y Y \to \operatorname{Stab}_{f(y)} X$  is an isomorphism of groups. A folklore theorem [KM, cf. Lemma 6.3] states that an étale stabilizer-

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