Michigan Math. J. 65 (2016), 389–412

Strong Regular Embeddings of Deligne–Mumford Stacks and Hypertoric Geometry

Dan Edidin

Abstract. We introduce the notion of *strong regular embeddings* of Deligne–Mumford stacks. These morphisms naturally arise in the related contexts of generalized Euler sequences and hypertoric geometry.

1. Introduction

Let G be a finite group acting on affine schemes $X = \operatorname{Spec} A$ and $Y = \operatorname{Spec} B$, and let $f: Y \to X$ be a G-equivariant morphism. Since f is G-equivariant, there is an induced map of invariant subrings $A^G \to B^G$ corresponding to a morphism of quotients $g: Y/G \to X/G$. Certain algebro-geometric properties of the morphism f (typically related to finiteness) are automatically preserved by the morphism g. For example, if f is finite, then the induced morphism of quotients $g: Y/G \to X/G$ is also finite. Likewise, if $|G|$ is a unit in $\operatorname{Spec} B$ and $f: Y \to X$ is a closed embedding, then $g: Y/G \to X/G$ is as well.

On the other hand, many properties of the morphism f will not descend. If f is flat or smooth, the induced morphism of quotients need not be. Instead, we can impose additional conditions on the actions of G on X and Y to ensure that a property of morphisms of schemes does descend to the quotient. Two obvious conditions that suffice are that G act freely or that G act trivially on both spaces.

Note, however, that these conditions are not necessary. For example, if $Y = X \times Z$ and G acts trivially on Z, then $Y/G = X/G \times Z$, so the flat projection $Y \to X$ descends to a flat projection $Y/G \to X/G$, and the diagram

$$\begin{array}{ccc} Y & \longrightarrow & X \\ \downarrow & & \downarrow \\ Y/G & \longrightarrow & X/G \end{array}$$

is Cartesian.

This is an example of a *stabilizer preserving morphism,* meaning that for every point $y \in Y$, the map of stabilizers $\operatorname{Stab}_y Y \to \operatorname{Stab}_{f(y)} X$ is an isomorphism of groups. A folklore theorem [KM, cf. Lemma 6.3] states that an étale stabilizer-

Received April 15, 2015. Revision received December 21, 2015.
The author's research was supported in part by Simons collaboration grant 315460.