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On Minimal Log Discrepancies on Varieties with Fixed Gorenstein Index

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ABSTRACT. We generalize the rationality theorem of the accumulation points of log canonical thresholds, which was proved by Hacon, McKernan, and Xu. Further, we apply the rationality to the ACC problem on the minimal log discrepancies. We study the set of log discrepancies on varieties with fixed Gorenstein index. As a corollary, we prove that the minimal log discrepancies of three-dimensional canonical pairs with fixed coefficients satisfy the ACC.

1. Introduction

The minimal log discrepancy (mld for short) was introduced by Shokurov in order to reduce the conjecture of terminations of flips to a local problem about singularities. Recently, this has been a fundamental invariant in the minimal model program. There are two conjectures on mlds, the ACC (ascending chain condition) conjecture and the LSC (lower semicontinuity) conjecture. Shokurov [22] showed that these two conjectures imply the conjecture of terminations of flips.

In this paper, we consider the ACC conjecture. For an $\mathbb{R}$-divisor D and a subset $I \subset \mathbb{R}$, we write $D \in I$ when all the nonzero coefficients of D belong to I. Further, for a subset $I \subset \mathbb{R}$, we say that I satisfies *the ascending chain condition* (resp. *the descending chain condition*) when there is no infinite increasing (resp. decreasing) sequence $a_i \in I$; *ACC* (resp. *DCC*) stands for the ascending chain condition (resp. the descending chain condition).

CONJECTURE 1.1 (ACC conjecture [21, Conj. 4.2]). *Fix $d \in \mathbb{Z}_{>0}$ and a subset $I \subset [0, 1]$ that satisfies the DCC. Then the set*

$$A(d, I) := \{\mathrm{mld}_x(X, \Delta) \mid (X, \Delta) \text{ is a log pair}, \dim X = d, \Delta \in I, x \in X\}$$

satisfies the ACC, where x is a closed point of X.

We are mainly interested in the case where I is a finite set. This is because the ACC conjecture for an arbitrary finite set I and the LSC conjecture imply the termination of flips [22].

The ACC conjecture is known for $d \leq 2$ by Alexeev [1] and Shokurov [20], and for toric pairs by Ambro [3]. Kawakita [11] proved the ACC conjecture on the interval $[1, 3]$ for three-dimensional smooth varieties. Further, Kawakita [10] proved that the ACC conjecture is true for a fixed variety X and a finite set I.

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