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Carleson Measures and Toeplitz Operators for Weighted Bergman Spaces on the Unit Ball

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Abstract. We obtain some new characterizations on Carleson measures for weighted Bergman spaces on the unit ball involving product of functions. For these, we characterize bounded and compact Toeplitz operators between weighted Bergman spaces. The results are applied to characterize bounded and compact extended Cesàro operators and pointwise multiplication operators. The results are new even in the case of the unit disk.

1. Introduction

Let $\mathbb{C}^n$ denote the Euclidean space of complex dimension n. For any two points $z = (z_1, \dots, z_n)$ and $w = (w_1, \dots, w_n)$ in $\mathbb{C}^n$, we write $\langle z, w\rangle = z_1\bar{w}_1 + \cdots + z_n\bar{w}_n$, and $|z| = \sqrt{\langle z, z\rangle} = \sqrt{|z_1|^2 + \cdots + |z_n|^2}$. Let $\mathbb{B}_n = \{z \in \mathbb{C}^n : \ |z| < 1\}$ be the unit ball in $\mathbb{C}^n$. Let $H(\mathbb{B}_n)$ be the space of all holomorphic functions on the unit ball $\mathbb{B}_n$. Let dv be the normalized volume measure on $\mathbb{B}_n$ such that $v(\mathbb{B}_n) = 1$. For $0 < p < \infty$ and $-1 < \alpha < \infty$, let $L^{p,\alpha} := L^p(\mathbb{B}_n, dv_\alpha)$ denote the weighted Lebesgue spaces that contain measurable functions f on $\mathbb{B}_n$ such that

$$\|f\|_{p,\alpha} = \left(\int_{\mathbb{B}_n} |f(z)|^p \, dv_\alpha(z)\right)^{1/p} < \infty,$$

where $dv_\alpha(z) = c_\alpha(1 - |z|^2)^\alpha \, dv(z)$, and c_α is the normalized constant such that $v_\alpha(\mathbb{B}_n) = 1$. We also denote by $A^p_\alpha = L^p(\mathbb{B}_n, dv_\alpha) \cap H(\mathbb{B}_n)$ the weighted Bergman space on $\mathbb{B}_n$, with the same norm. If $\alpha = 0$, then we simply write them as $L^p(\mathbb{B}_n, dv)$ and A^p, respectively, and $\|f\|_p$ for the norm of f in these spaces.

Let μ be a positive Borel measure on $\mathbb{B}_n$. For $\lambda > 0$ and $\alpha > -1$, we say that μ is a (λ, α)-*Bergman–Carleson measure* if for any two positive numbers p and q with $q/p = \lambda$, there is a positive constant $C > 0$ such that

$$\int_{\mathbb{B}_n} |f(z)|^q \, d\mu(z) \le C\|f\|^q_{p,\alpha}$$

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