Michigan Math. J. 64 (2015), 263–277

Branching of Automorphic Fundamental Solutions

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Abstract. Automorphic fundamental solutions and, more generally, solutions of automorphic differential equations play a key role in the Diaconu–Garrett–Goldfeld prescription for spectral identities involving moments of L-functions [4; 5; 6] as well as other applications, including an explicit formula relating the number of lattice points in a symmetric space to the automorphic spectrum [2]. In this paper we discuss two cases in which the automorphic fundamental solution exhibits branching: pathwise meromorphic continuations may differ by a term involving an Eisenstein series.

1. Introduction

Solutions of automorphic differential equations underlie the Diaconu–Garrett–Goldfeld prescription for spectral identities involving second moments for arbitrary Rankin–Selberg integral representations of L-functions [6]. This prescription is a vast generalization of the constructions of moment identities in their earlier papers, from which they extracted subconvex bounds for GL_2 automorphic L-functions [7; 8; 4; 5]. Essential to their prescription is a Poincaré series, whose data was originally constructed in imitation of Good's kernel [9]; characterizing the Poincaré series as the solution to an automorphic differential equation allows generalization from GL_2 to higher rank. The automorphic spectral expansion of such a Poincaré series is heuristically immediate and can be legitimized using automorphic Sobolev theory, developed in [2]. In general, explicit geometric expressions for solutions of automorphic differential equations are very difficult to obtain; see [3] for some examples, including the automorphic fundamental solution that is used in the lattice-point counting application in [2] and is suitable for constructing moment identities for $GL_n(\mathbb{C}) \times GL_n(\mathbb{C})$ Rankin–Selberg L-functions. Superficially, the spectral expansion of the automorphic fundamental solution appears to be invariant under a transformation of an auxiliary complex parameter w, but a closer look reveals, in certain cases, *branching* in w, eliminating the possibility of a straightforward functional equation.

Let G be a semisimple Lie group, K its maximal compact subgroup, and Γ a discrete subgroup. Consider the solution of the following differential equation on the arithmetic quotient $X = \Gamma\backslash G/K$:

$$(\Delta - \lambda_w)^{\nu} u_w = \delta_{z_o},$$

Received January 9, 2014. Revision received November 20, 2014.

The author was partially supported by a research grant from the University of St. Thomas.