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Cycle-Level Products in Equivariant Cohomology of Toric Varieties

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Abstract. In this paper, we define an action of the group of equivariant Cartier divisors on a toric variety X on the equivariant cycle groups of X, arising naturally from a choice of complement map on the underlying lattice. If X is nonsingular, then this gives a lifting of the multiplication in equivariant cohomology to the level of equivariant cycles. As a consequence, one naturally obtains an equivariant cycle representative of the equivariant Todd class of any toric variety. These results extend to equivariant cohomology the results of [Th] and [PT]. In the case of a complement map arising from an inner product, we show that the equivariant cycle Todd class obtained from our construction is identical to the result of the inductive, combinatorial construction of Berline and Vergne [BV1; BV2]. In the case of arbitrary complement maps, we show that our Todd class formula yields the local Euler–Maclaurin formula introduced in [GP].

1. Introduction

1.1. Overview

Intersection theory on a nonsingular algebraic variety provides a natural intersection product of cycles modulo rational equivalence. One might wonder in what circumstances there is a reasonable lifting of this product to the level of cycles, so that any two cycles on X on can be multiplied to produce a well-defined cycle on X in a natural way that respects rational equivalence. If the cycles intersect properly, then there is a natural product, but if the intersection is not proper, then one must settle for knowing the product only as a cycle modulo rational equivalence. More generally, for arbitrary (possibly singular) algebraic varieties, intersection theory provides an action of the Picard group of an arbitrary algebraic variety on the Chow groups of the variety. One can ask if there is a natural lifting of the action of the Picard group to the level of algebraic cycles. For toric varieties, such an action was constructed in [Th]. The action depends on the choice of a complement map, which is a certain global choice of linear subspaces. (See Section 1.2 for details.)

One of the motivations for a cycle-level intersection theory is that it leads naturally to a cycle expression for the Todd class of a toric variety. Indeed, the Todd class of a nonsingular toric variety has a well-known expression as a product of

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