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On the Supersingular $K3$ Surface in Characteristic 5 with Artin Invariant 1

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Dedicated to Professor Igor V. Dolgachev on the occasion of his 70th birthday.

Abstract. We present three interesting projective models of the supersingular $K3$ surface X in characteristic 5 with Artin invariant 1. For each projective model, we determine smooth rational curves on X with the minimal degree and the projective automorphism group. Moreover, by using the superspecial Abelian surface we construct six sets of 16 disjoint smooth rational curves on X and show that they form a beautiful configuration.

1. Introduction

Let Y be a $K3$ surface defined over an algebraically closed field k, and $\rho(Y)$ the Picard number of Y. Then it is well known that $1 \leq \rho(Y) \leq 20$ or $\rho(Y) = 22$. The last case $\rho(Y) = 22$ occurs only when k is of positive characteristic. A $K3$ surface is called *supersingular* if its Picard number is 22. Let Y be a supersingular $K3$ surface in characteristic $p \geq 3$. Let S_Y denote its Néron–Severi lattice, and let $S_Y^\vee$ be the dual of S_Y. Then Artin [1] proved that $S_Y^\vee / S_Y$ is a p-elementary Abelian group of rank 2σ, where σ is an integer such that $1 \leq \sigma \leq 10$. This integer σ is called the *Artin invariant* of Y. It is known that the isomorphism class of S_Y depends only on p and σ (Rudakov and Shafarevich [26]). On the other hand, supersingular $K3$ surfaces with Artin invariant σ form a $(\sigma - 1)$-dimensional family, and a supersingular $K3$ surface with Artin invariant 1 in characteristic p is unique up to isomorphisms (Ogus [24; 25], Rudakov and Shafarevich [26]).

Supersingular $K3$ surfaces in *small* characteristic p with Artin invariant 1 are especially interesting because big finite groups act on them by automorphisms. (See Dolgachev and Keum [11].) For example, the group $\mathrm{PGL}(3, \mathbb{F}_4) \ltimes \mathbb{Z}/2\mathbb{Z}$ in case $p = 2$ or $\mathrm{PGU}(4, \mathbb{F}_9)$ in case $p = 3$ acts on the $K3$ surface by automorphisms. Moreover, these $K3$ surfaces contain a finite set of smooth rational curves on which the above group acts as symmetries. For example, in case $p = 2$, there exist 42 smooth rational curves that form a (21_5)-configuration (see Dolgachev and Kondo [12], Katsura and Kondo [16]). In case $p = 3$, the Fermat quartic surface is a supersingular $K3$ surface with Artin invariant 1, and it contains 112 lines (e.g., Katsura and Kondo [15], Kondo and Shimada [19]).

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