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New Examples of Constant Mean Curvature Surfaces in $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$

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Abstract. We construct nonzero constant mean curvature H surfaces in the product spaces $\mathbb{S}^2 \times \mathbb{R}$ and $\mathbb{H}^2 \times \mathbb{R}$ by using suitable conjugate Plateau constructions. The resulting surfaces are complete, have bounded height, and are invariant under a discrete group of horizontal translations. A one-parameter family of unduloid-type surfaces is produced in $\mathbb{S}^2 \times \mathbb{R}$ for any $H > 0$ (some of which are compact) and in $\mathbb{H}^2 \times \mathbb{R}$ for any $H > 1/2$ (which are shown to be properly embedded bigraphs). Finally, we give a different construction in $\mathbb{H}^2 \times \mathbb{R}$ for $H = 1/2$, giving surfaces with the symmetries of a tessellation of $\mathbb{H}^2$ by regular polygons.

1. Introduction

In 1970, Lawson [Law70] established a celebrated correspondence between simply connected minimal surfaces in a space form $M^3(\kappa)$ (with constant curvature κ) and constant mean curvature (CMC) H surfaces in the space $M^3(\kappa - H^2)$. This result motivated the construction of two doubly periodic constant mean curvature one surfaces in the Euclidean 3-space. The procedure used to obtain such examples is known as the *conjugate Plateau construction* and has become a fruitful method to obtain constant mean curvature surfaces in space forms (e.g., see [KPS88; K89; GB93; Po94]). We summarize the steps of this construction as follows:

(1) Solve the Plateau problem in a geodesic polygon in $M^3(\kappa)$.
(2) Consider the *conjugate* CMC H surface in $M^3(\kappa - H^2)$, whose boundary lies on some planes of symmetry since the initial surface is bounded by geodesic curves (see [K89, Section 1]).
(3) Reflect the resulting surface across its edges to get a complete constant mean curvature H surface in $M^3(\kappa - H^2)$.

The key property of this method is that a *geodesic curvature line in the initial surface becomes a planar line of symmetry in the conjugate one.* This is crucial in order to extend by reflection the conjugate piece to a complete constant mean curvature surface. Hence, it is important to cleverly choose the appropriate geodesic polygon once the desired symmetries in the target surface have been fixed.

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