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Branched Spherical CR Structures on the Complement of the Figure-Eight Knot

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Abstract. We obtain a branched spherical CR structure on the complement of the figure-eight knot whose holonomy representation was given in [4]. There are essentially two boundary unipotent representations from the complement of the figure-eight knot into $\mathbf{PU}(2, 1)$, which we call ρ_1 and ρ_2. We make explicit some fundamental differences between these two representations. For instance, seeing the figure-eight knot complement as a surface bundle over the circle, the behavior of the fundamental group of the fiber under the representation is a key difference between ρ_1 and ρ_2.

1. Introduction

The three-dimensional sphere contained in $\mathbb{C}^2$ inherits a Cauchy–Riemann structure as the boundary of the complex two-ball. Three-dimensional manifolds locally modeled on the sphere then are called spherical CR manifolds and have been studied since Cartan [2]. Spherical CR structures appear naturally as quotients of an open subset of the three-dimensional sphere by a subgroup of the CR automorphism group (denoted $\mathbf{PU}(2, 1)$) (see [8; 7] and [12] for a recent introduction).

The irreducible representations of the fundamental group of the complement of the figure-eight knot into $\mathbf{PU}(2, 1)$ with unipotent boundary holonomy were obtained in [4]. To obtain such representations, the existence of a developing map obtained from the 0-skeleton of an ideal triangulation is imposed. Solution of a system of algebraic equations gives rise to a set of representations of $\Gamma = \pi_1(M)$, the fundamental group of the complement of the figure-eight knot with parabolic peripheral group.

Up to precomposition with automorphisms of Γ, there exist two irreducible representations into $\mathbf{PU}(2, 1)$ with unipotent boundary holonomy (see [3]). Following [4], we call them ρ_1 and ρ_2. In [4], we showed that ρ_1 could be obtained from a branched spherical CR structure on the knot complement. Moreover, this representation is not the holonomy of a uniformizable structure since the limit set is the full sphere S^3.

In this paper, we analyze ρ_2 and show that it is also obtained as the holonomy of a branched structure in Theorem 12 in Section 6. The branched locus in the complement of the figure-eight knot is a segment with end points in the knot

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