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Weighted Multilinear Square Function Bounds

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Abstract. We study the boundedness of Littlewood–Paley–Stein square functions associated to multilinear operators. We prove weighted Lebesgue space bounds for square functions under relaxed regularity and cancellation conditions that are independent of weights, which is a new result even in the linear case. For a class of multilinear convolution operators, we prove necessary and sufficient conditions for weighted Lebesgue space bounds. Using extrapolation theory, we extend weighted bounds in the multilinear setting for Lebesgue spaces with index smaller than one.

1. Introduction

Given a function $\psi : \mathbb{R}^n \to \mathbb{C}$, define $\psi_t(x) = t^{-n}\psi(t^{-1}x)$ and the associated Littlewood–Paley–Stein-type square function

$$g_\psi(f) = \left(\int_0^\infty |\psi_t * f|^2 \frac{dt}{t}\right)^{1/2}. \tag{1.1}$$

These convolution-type square functions were introduced by Stein in the 1960s, see for example [32] or [33], and have been studied extensively since then, including classical works by Stein [32], Kurtz [24], Duoandikoetxea and Rubio de Francia [11], and more recent works by Duoandikoetxea and Seijo [12], Cheng [4], Sato [30], Duoandikoetxea [10], Wilson [35], Lerner [25], and Cruz-Uribe, Martell, and Perez [8]. Of particular relevance to this work are [24; 12; 30; 35; 8] and [25], which prove bounds for g_ψ on weighted Lebesgue spaces under various conditions on ψ. Nonconvolution variants of (1.1) were studied by Carleson [3], David, Journé, and Semmes [9], Christ and Journé [5], Semmes [31], Hofmann [22; 21], and Auscher [2], where they replaced the convolution $\psi_t * f(x)$ with

$$\Theta_t f(x) = \int_{\mathbb{R}^n} \theta_t(x,y) f(y)\, dy.$$

In [9] and [31], the authors proved L^p bounds for Littlewood–Paley–Stein square functions associated to Θ_t when $\Theta_t(b) = 0$ for some para-accretive function b. In [22; 21], this type of mean zero assumption is replaced by a local cancellation testing condition on dyadic cubes. In [3; 5] and [2], the authors replace the mean zero assumption with a Carleson measure condition for θ_t to prove L^2 bounds for the square function. The work of Carleson [3] was phrased as a characterization

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