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# Common Boundary Values of Holomorphic Functions for Two-Sided Complex Structures

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Abstract. Let $\Omega_1, \Omega_2$ be two disjoint open sets in $\mathbf{R}^{2n}$ whose boundaries share a smooth real hypersurface $M$ as a relatively open subset. Assume that $\Omega_i$ is equipped with a complex structure $J^i$ that is smooth up to $M$. Suppose that at each point $x \in M$ there is a vector $v \in T_x M$ such that $J_x^1 v$ and $J_x^2 v$ are in the same connected component of $T_x \mathbf{R}^{2n} \setminus T_x M$. If $f$ is holomorphic with respect to both structures in the open sets and continuous on $\Omega_1 \cup M \cup \Omega_2$, then $f$ must be smooth on the union $\Omega_1 \cup M$. Although the result, as stated, is far more meaningful for integrable structures, our methods make it much more natural to deal with the general almost complex structures without the integrability condition. The result is therefore proved in the framework of almost complex structures.

## 1. Introduction

In this paper, we study the regularity of boundary values of two functions that are holomorphic with respect to two complex structures defined on two domains separated by a real hypersurface. We are interested in the situation where the two functions have the same continuous boundary values on the hypersurface. Notice that the regularity becomes an interior property when the two structures are the restriction of the same complex structure, which is well established by the Newlander–Nirenberg theorem; our results are concerned with a pair of distinct structures. To highlight the relevance of our problem to yet another classical regularity problem, we recall the edge-of-the-wedge theorem, which deals with two holomorphic functions on two wedges in $\mathbf{C}^n$ that have the same appropriate boundary values on the edge. Under suitable assumptions on the wedges, the theorem concludes that both functions are actually the restriction of the same holomorphic function defined on a domain containing the edge. The edge-of-the-wedge theorem, originally due to Bogolyubov, has been extended in great generality by many authors. For instance, see Rudin [20] and references therein, Morimoto [15], Pinčuk [18], Bedford [2], Straube [23], Rosay [19], Forstnerič [8], and Eastwood and Graham [7]. Despite some similarity between the classical edge-of-the-wedge theorem and our results, the study of the regularity in this paper apparently gives rise to a new

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