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Lower Bound for the Geometric Type from a Generalized Estimate in the $\bar\partial$-Neumann Problem – a New Approach by Peak Functions

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1. Introduction

In a series of seminal papers in the Annals of Mathematics [Cat83; Cat87], Catlin proved the equivalence of the finite type of a boundary (cf. [D'A82]) with the existence of a subelliptic estimate for the $\bar\partial$-Neumann problem by triangulating through the t^ε-property (see below)

(i) finite type $m \Rightarrow t^\varepsilon$-property with $\varepsilon = m^{-n^2 m^{n^2}}$;
(ii) t^ε-property $\Rightarrow \varepsilon$-subelliptic estimate;
(iii) ε-subelliptic estimate $\Rightarrow$ finite type m for $m \le \frac{1}{\varepsilon}$.

Here, the t^ε-property of a boundary $b\Omega$ is a special case of a more general "f-property" defined as follows. For a smooth strictly increasing function $f : [1+\infty) \to [1,+\infty)$ with $f(t) \le t^{1/2}$, the f-property at z_o means the existence of a neighborhood U of z_o, of constants C_1, C_2, and of a family of functions $\{\phi_\delta\}$ such that

1) ϕ_δ are plurisubharmonic and C^2 on U, and $-1 \le \phi_\delta \le 0$;
2) $\partial\bar\partial\phi_\delta \ge C_1 f(\delta^{-1})^2 Id$ and $|D\phi_\delta| \le C_2\delta^{-1}$ for any $z \in U \cap \{z \in \Omega\colon -\delta < r(z) < 0\}$, where r is a defining function of Ω.

The results in steps (ii) and (iii) were generalized in [KZ10; KZ12]. In particular, in [KZ10] it was shown that the f-property implies an f-estimate for any f, and in [KZ12] that an f-estimate with $\frac{f}{\log} \to \infty$ at ∞ implies that the type along a complex analytic variety has a lower bound with the rate G with

$$G(\delta) = \left(\left(\frac{f}{\log}\right)^*(\delta^{-1})\right)^{-1}, \tag{1.1}$$

where the superscript * denotes the inverse function. Combining the above results, we obtain the following:

Theorem 1.1 (Catlin [Cat83; Cat87]; Khanh and Zampieri [KZ10; KZ12]). *Let Ω be a pseudoconvex domain in $\mathbb{C}^n$ with C^∞-smooth boundary $b\Omega$, and z_o be a boundary point. Assume that the f-property holds at z_o with $\frac{f}{\log} \nearrow \infty$ as $t \to \infty$.*

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