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Embedded Triply Periodic Zero Mean Curvature Surfaces of Mixed Type in Lorentz–Minkowski 3-Space

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1. Introduction

In any robust surface theory, it is essential to have a large collection of interesting examples. An interesting class of surfaces to study is the zero-mean curvature surfaces of mixed type in the Lorentz–Minkowski three-space $\mathbf{R}^3_1$, which, roughly speaking, are smooth surfaces of mixed causal type with mean curvature, wherever well-defined, equal to zero.

Several authors have found examples of such surfaces [12; 6; 14; 11; 3; 4], all having simple topology. The main goal of this article is to provide a concrete example of a family of such surfaces with nontrivial topology.

The motivation for the method of our construction is the fact that fold singularities of spacelike maximal surfaces have real analytical extensions to timelike minimal surfaces (see [6; 7; 8; 11; 9; 4], and, especially, [4], which is an expository article on this subject). Main ingredients are the spacelike maximal analogues in $\mathbf{R}^3_1$ of the Schwarz P surfaces and the Schwarz D surfaces in $\mathbf{R}^3$, which were remarked upon in the previous work [5] by the authors. The Schwarz P-type maximal surfaces admit cone-like singularities, whereas the Schwarz D-type maximal surfaces admit fold singularities (cf. Figure 1). By extending the Schwarz D-type (spacelike) maximal surfaces to timelike minimal surfaces, we obtain the following main result of this article.

Theorem A. *The one-parameter family of Schwarz D-type spacelike maximal surfaces* $\{X_a\}_{0<a<1}$ *has a unique analytic extension*

$$\tilde{X}_a : \Sigma_a \to \mathbf{R}^3_1/\Gamma_a \quad (0 < a < 1)$$

to embedded zero-mean-curvature surfaces, where $\mathbf{R}^3_1/\Gamma_a$ *is a torus given by a suitable three-dimensional lattice* Γ_a*, and* Σ_a *is a closed orientable 2-manifold of genus three* (*cf. Figure 2*).

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