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Remarks on the Metric Induced by the Robin Function II

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1. Introduction

Let D be a C^∞-smoothly bounded domain in $\mathbf{C}^n$ $(n \geq 2)$. For $p \in D$, let $G(z,p)$ be the Green function for D with pole at p associated to the standard Laplacian

$$\Delta = 4\sum_{i=1}^{n} \frac{\partial^2}{\partial z_i \partial \bar{z}_i}$$

on $\mathbf{C}^n \approx \mathbf{R}^{2n}$. Then $G(z,p)$ is the unique function of $z \in D$ satisfying the conditions that $G(z,p)$ is harmonic on $D \setminus \{p\}$, $G(z,p) \to 0$ as $z \to \partial D$, and $G(z,p) - |z-p|^{-2n+2}$ is harmonic near p. Thus

$$\Lambda(p) = \lim_{z\to p}(G(z,p) - |z-p|^{-2n+2})$$

exists and is called the *Robin constant* for D at p. The function

$$\Lambda\colon p \to \Lambda(p)$$

is called the *Robin function* for D.

The Robin function for D is negative and real-analytic, and it tends to $-\infty$ near ∂D (see [10]). Furthermore, if D is pseudoconvex then, by a result of Levenberg and Yamaguchi [7], $\log(-\Lambda)$ is a strongly plurisubharmonic function on D. Therefore,

$$ds^2 = \sum_{\alpha,\beta=1}^{n} \frac{\partial^2 \log(-\Lambda)}{\partial z_\alpha \partial \bar{z}_\beta}\, dz_\alpha \otimes d\bar{z}_\beta$$

is a Kähler metric on D, which is called the Λ-*metric*. Recall that the holomorphic sectional curvature of ds^2 at $z \in D$ along the direction $v \in \mathbf{C}^n$ is given by

$$R(z,v) = \frac{R_{\alpha\bar{\beta}\gamma\bar{\delta}} v^\alpha \bar{v}^\beta v^\gamma \bar{v}^\delta}{g_{\alpha\bar{\beta}} v^\alpha \bar{v}^\beta};$$

here

$$R_{\alpha\bar{\beta}\gamma\bar{\delta}} = -\frac{\partial^2 g_{\alpha\bar{\beta}}}{\partial z_\gamma \partial \bar{z}_\delta} + g^{\nu\bar{\mu}} \frac{\partial g_{\alpha\bar{\mu}}}{\partial z_\gamma} \frac{\partial g_{\nu\bar{\beta}}}{\partial \bar{z}_\delta}$$

are the components of the curvature tensor,

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