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Coherence and Negative Sectional Curvature in Complexes of Groups

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1. Introduction

A group G is *coherent* if finitely generated subgroups are finitely presented. A group G is *locally quasiconvex* if each finitely generated subgroup is quasiconvex. A subgroup H of G is *quasiconvex* if there is a constant L such that every geodesic in the Cayley graph of G that joins two elements of H lies in an L-neighborhood of H. While L depends upon the choice of Cayley graph, it is well known that the quasiconvexity of H is independent of the finite generating set when G is hyperbolic. As quasiconvex subgroups are finitely presented, it is clear local quasiconvexity implies coherence.

The class of coherent groups includes fundamental groups of compact 3-manifolds by a result of Scott [13], mapping tori of free group automorphisms by work of Feighn and Handel [3], and 1-relator groups with sufficient torsion by McCammond and Wise [11]. In contrast, the class of locally quasiconvex groups is substantially smaller. It includes fundamental groups of infinite-volume hyperbolic 3-manifolds by a result of Thurston ([12, Prop. 7.1] or [9, Thm. 3.11]), and there are criteria for local quasiconvexity for certain classes of small cancellation groups [8; 11].

Criteria for proving coherence and local quasiconvexity of groups acting freely on simply connected 2-complexes was introduced in [15] based on a notion of combinatorial sectional curvature. These methods do not apply on groups with torsion unless they are known to be virtually torsion free. It is an open question whether negatively curved groups are virtually torsion free [5].

In this paper we revisit the notion of combinatorial sectional curvature. We provide criteria for coherence and local quasiconvexity of groups acting properly and cocompactly on simply connected 2-complexes. This extends the methods in [15] to groups with torsion. Our extension of these results involves a generalization of the notion of sectional curvature and an extension of the combinatorial Gauss–Bonnet theorem to complexes of groups, and it surprisingly requires the use of ℓ^2-Betti numbers.

We revisit the following notion of sectional curvature in Section 3.

Definition 1.1 (Sectional Curvature $\leq \alpha$). An *angled 2-complex X* is a combinatorial 2-complex with an assignment of a real number to each corner of each

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