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Point Pushing, Homology, and the Hyperelliptic Involution

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1. Introduction

Let S_g denote a closed, connected, orientable surface of genus g. The hyperelliptic Torelli group $\mathcal{SI}(S_g)$ is the subgroup of the mapping class group $\mathrm{Mod}(S_g)$ consisting of all elements that act trivially on $H_1(S_g; \mathbb{Z})$ and that commute with the isotopy class of some fixed hyperelliptic involution $s\colon S_g \to S_g$, that is, any order 2 homeomorphism acting by $-I$ on $H_1(S_g; \mathbb{Z})$. Every hyperelliptic involution of S_g is conjugate to the one shown in Figure 1.

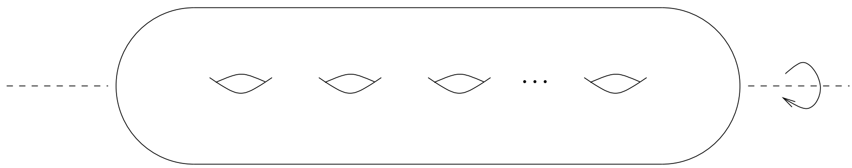

Figure 1 Rotation by π about the indicated axis is a hyperelliptic involution

The group $\mathcal{SI}(S_g)$ arises in algebraic geometry in the following context. Let $\mathcal{T}(S_g)$ denote the cover of the moduli space of Riemann surfaces corresponding to the Torelli subgroup $\mathcal{I}(S_g)$ of $\mathrm{Mod}(S_g)$. The period mapping is a function from $\mathcal{T}(S_g)$ to the Siegel upper half-space of rank g and is a 2-fold branched cover onto its image. The branch locus is the set of hyperelliptic points of $\mathcal{T}(S_g)$, the union of the fixed sets for the actions of the various hyperelliptic involutions on $\mathcal{T}(S_g)$. These fixed sets are pairwise disjoint, and the fundamental group of each component is isomorphic to $\mathcal{SI}(S_g)$. Because of this, $\mathcal{SI}(S_g)$ is related, for example, to the topological Schottky problem; see [9, Prob. 1].

A basic tool in the theory of mapping class groups is the Birman exact sequence. This sequence relates the mapping class group of a surface with marked points to the mapping class group of the surface obtained by forgetting the marked points; see Section 3. This is a key ingredient for performing inductive arguments on the

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