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Derived Categories of Toric Varieties II

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1. Introduction

This paper supplements the first part of the series [5], where we considered the semi-orthogonal decompositions of the derived categories with respect to the toric minimal model program. We proved that the semi-orthogonal complements for divisorial contractions and flips have full exceptional collections.

A. Ishii and K. Ueda pointed out that the divisorial extractions, another important class of birational transformations, had not yet been treated. We did not consider toric birational morphisms $f: X \to Y$ between $\mathbf{Q}$-factorial projective toric varieties whose exceptional locus is a prime divisor E such that $K_X + eE = f^*K_Y$ with $e > 0$. We remark that (i) if $e < 0$ then f is a divisorial contraction (which is already treated in [5]) and (ii) if $e = 0$ then f is a log crepant morphism and we have a derived equivalence [4]. So we consider this case in Section 1 and prove that the semi-orthogonal complement again has a full exceptional collection. This result is rather remarkable when one considers that, for the fully faithful embedding functors between derived categories, the directions are opposite in the cases $e > 0$ and $e < 0$.

We also correct certain notation in [5]—namely, in the paragraph before Remark 5.1—in response to a remark of Ishii and Ueda. Thus we write $j_{1*}j_2^*$ instead of j^* because there is no morphism of stacks over a morphism of schemes $D \to X$. However, we do establish a fliplike diagram $\mathcal{D} \leftarrow \tilde{\mathcal{D}} \to \mathcal{X}$; see the text between Lemmas 2 and 3 (to follow).

In Section 2 we answer a question posed by S. Okawa at the Chulalongkorn University conference. We prove that the number of Fourier–Mukai partners of a $\mathbf{Q}$-factorial projective toric variety is finite, confirming a conjecture related to the finiteness conjecture of the minimal models (see [2]).

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2. Divisorial Extraction

We proved in [4] that the minimal model program in the category of toroidal varieties yields semi-orthogonal decompositions of derived categories. In [5] we

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