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A New Existence Proof of the Monster by VOA Theory

Robert L. Griess Jr. & Ching Hung Lam

1. Introduction

We define a finite group G to be of *Monster type* if it has an involution z whose centralizer $C_G(z)$ has the form $2^{1+24}\mathrm{Co}_1$ and is 2-constrained (i.e., satisfies $\langle z\rangle = C_G(O_2(C_G(z)))$ and if z is conjugate to an element in $C_G(z)\setminus\{z\}$. A short argument proves that such a G must be simple (e.g., see [21; 46]). We use the abbreviation VOA for *vertex operator algebra* [17].

This paper gives a new and relatively direct existence proof of a group of Monster type. Our methods depend on vertex operator algebra representation theory and are free of many special calculations that traditionally occur in theory of the Monster. Most of this article is dedicated to explaining how existing VOA theory applies.

In fact, a group of Monster type is unique up to isomorphism [23], so the group we construct here can be called "the" Monster, the group constructed in [21]. To avoid specialized finite group theory in this article, we work with a group of Monster type and refer to [23] for uniqueness.

Our basic strategy is described briefly in the next paragraph. It was inspired by the article of Miyamoto [35], which showed how to make effective use of *simple current modules and extensions.* Later in this Introduction, we sketch these important concepts. In a sense, our existence proof is quite short. The hard group theory and case-by-case analysis of earlier proofs have essentially been eliminated.

In [41], Shimakura gives a variation of Miyamoto's construction. He takes $(V_{EE_8}^+)^3$ and builds a candidate V for the Moonshine VOA using the theory of simple current extensions (a short account is given in Section 2.1). His treatment is more direct and shorter than Miyamoto's. Moreover, his method furnishes a large subgroup of $\mathrm{Aut}(V)$. From this subgroup, we take a certain involution and analyze V^+, V^-, its fixed point VOA and its negated space on V, respectively. We can recognize V^+ as a Leech lattice-type VOA. The group $\mathrm{Aut}(V^+)$ and its extension (by projective representations) to irreducibles of the fixed point VOA are understood. One of these irreducibles is V^-. We thereby get a new subgroup of $\mathrm{Aut}(V)$, which has the shape $2^{1+24}\mathrm{Co}_1$ and is moreover isomorphic to the centralizer of a 2-central involution in the Monster. These two subgroups of $\mathrm{Aut}(V)$ generate the larger group $\mathrm{Aut}(V)$, which we then prove is a finite group of Monster type.

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