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First-Order Local Invariants of Stable Maps from 3-Manifolds to $\mathbb{R}^3$

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1. Introduction

The study of topological invariants of stable maps has been one of the main problems in singularity theory in the last decades. An important question concerns the global study of stable maps between 3-manifolds. This is a very difficult problem when one tries to study it in a general setting. Little is known from the global viewpoint; even in the particular case of stable maps from the 3-sphere to Euclidean 3-space. In [19], Vassiliev introduced a method to define isotopy invariants for stable maps in a fairly general context that has proved to be useful in the case of knots (considered as stable maps from the circle to 3-space; see [18]). Such invariants can be seen as locally constant functions on the considered subspace of stable maps, and the method is based on analyzing the structure of the discriminant set (subset of nonstable maps) in the total space of the maps in question. Vassiliev's techniques have also been used to study other subspaces of stable maps, such as immersed closed plane curves [1; 2], stable maps between surfaces [16], stable maps from surfaces into Euclidean 3-space [7], and stable maps from 3-manifolds to the plane [20]. This method, although it does not provide all the global invariants, does lead to an important set of invariants that contain all the information on multilocal behavior of the stable maps. These invariants are, in fact, related to the properties of the branch set (image of the singular set). To combine them with the global invariants related to the topology of the singular set in the source manifold (such as the graphs introduced in [8; 9; 10] for stable maps of surfaces into the plane) is to take a first step toward a global classification in each particular case of stable maps.

In order to apply this method to the case of stable maps from a closed 3-manifold to $\mathbb{R}^3$, we need to know all the codimension-1 and codimension-2 phenomena for maps between 3-manifolds (or from $\mathbb{R}^3$ to $\mathbb{R}^3$, from a local viewpoint) as well as the bifurcation diagrams for those of codimension 2. For this we have used the classification of corank-1 germs together with the information on bifurcation diagrams contained in the papers by Goryunov [6] and Marar and Tari [13]. A sufficient list of multigerms is obtained in Section 2.2 of this paper, and some of their bifurcation diagrams are calculated in the Appendix (Section 8). In Section 3 we obtain

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