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Cut Ideals of K_4-Minor Free Graphs Are Generated by Quadrics

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1. Introduction

In this paper we prove a conjecture of Sturmfels and Sullivant [12] about toric ideals used in algebraic statistics. A new connection between commutative algebra and statistics was made by Diaconis and Sturmfels [5] when they introduced the fundamental notion of Markov basis. To explain the connection, we use the first example from the Oberwolfach lectures on algebraic statistics by Drton, Strumfels, and Sullivant [7].

EXAMPLE 1.1. In a contingency table, both data and some marginals are reported. In Table 1, these marginals are the row and column sums. In order to test statistically the hypothesis that the verdicts are from a distribution independent of race, one must sample from a set of tables with the same marginals as Table 1. The usual way to sample is by a random walk on the set of tables with prescribed marginals, stopping when some test indicates that enough information has been collected. The nontrivial task is to find good steps (*Markov moves*) for the random walk, and here commutative algebra enters the picture.

Encode the numbers in Table 1 with monomials as in Table 2. The data entries in Table 2 are collected in the monomial $q_{11}^{19}q_{12}^{141}q_{21}^{17}q_{22}^{149} \in \mathbb{K}[q_{11}, q_{12}, q_{21}, q_{22}]$ and the marginal entries in the monomial $r_{1*}^{160}r_{2*}^{166}r_{*1}^{36}r_{*2}^{290} \in \mathbb{K}[r_{1*}, r_{2*}, r_{*1}, r_{*2}]$. The calculations that translate row and column sums into the algebraic setting are given by the ring homomorphism

Table 1 Data on death penalty verdicts [1, 5.2.2]

Defendant's race	Yes	No	Total
White	19	141	160
Black	17	149	166
Total	36	290	326

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