Michigan Math. J. 60 (2011), 331–354

Birational Invariants Defined by Lawson Homology

Wenchuan Hu

1. Introduction

In this paper, all varieties are defined over $\mathbb{C}$. Let X be an n-dimensional projective variety. The *Lawson homology* $L_pH_k(X)$ of p-cycles is defined by

$$L_pH_k(X) := \pi_{k-2p}(\mathcal{Z}_p(X)) \quad \text{for } k \geq 2p \geq 0,$$

where $\mathcal{Z}_p(X)$ is provided with a natural topology (see [F1; L1; Li1] for the quasi-projective case). For general background, the reader is referred to Lawson's survey paper [L2].

In [FM], Friedlander and Mazur showed that there are natural transformations, called *cycle class maps,*

$$\Phi_{p,k} \colon L_pH_k(X) \to H_k(X).$$

Definition 1.

$$\begin{aligned} L_pH_k(X)_{\text{hom}} &:= \ker\{\Phi_{p,k} \colon L_pH_k(X) \to H_k(X)\}; \\ T_pH_k(X) &:= \text{Image}\{\Phi_{p,k} \colon L_pH_k(X) \to H_k(X)\}; \\ T_pH_k(X, \mathbb{Q}) &:= T_pH_k(X) \otimes \mathbb{Q}. \end{aligned}$$

The *Griffiths group* of codimension q-cycles is defined to

$$\text{Griff}^q(X) := \mathcal{Z}^q(X)_{\text{hom}}/\mathcal{Z}^q(X)_{\text{alg}}$$

It was proved by Friedlander [F1] that, for any smooth projective variety X, $L_pH_{2p}(X) \cong \mathcal{Z}_p(X)/\mathcal{Z}_p(X)_{\text{alg}}$. Therefore

$$L_pH_{2p}(X)_{\text{hom}} \cong \text{Griff}_p(X),$$

where $\text{Griff}_p(X) := \text{Griff}^{n-p}(X)$.

It was shown in [FM, Sec. 7] that the subspaces $T_pH_k(X, \mathbb{Q})$ form a decreasing filtration,

$$\cdots \subseteq T_pH_k(X, \mathbb{Q}) \subseteq T_{p-1}H_k(X, \mathbb{Q}) \subseteq \cdots \subseteq T_0H_k(X, \mathbb{Q}) = H_k(X, \mathbb{Q}),$$

and that $T_pH_k(X, \mathbb{Q})$ vanishes if $2p > k$.

Received August 10, 2009. Revision received July 9, 2010.
This material is based upon work supported by the NSF under agreement no. DMS-0635607.