Michigan Math. J. 60 (2011), 289–312

Equisingularity of Families of Hypersurfaces and Applications to Mappings

KEVIN HOUSTON

1. Introduction

Notions of equisingularity for varieties date back many years, the modern era being started effectively by Zariski. Much work has been done in this area; see [2] for some recent significant and interesting results. A classical theorem of Teissier and of Briançon and Speder gives conditions for equisingularity of a family of complex hypersurfaces such that each member has an isolated singularity. In this case the family is called *Whitney equisingular* if the singular set of the variety formed by the family is a stratum in a Whitney stratification.

For any isolated hypersurface singularity we may associate a μ^*-*sequence*: The intersection of the Milnor fibre of the singularity and a generic i-plane passing through the singularity is homotopically equivalent to a wedge of spheres, the number of which is denoted μ^i. This is a sequence of analytic invariants.

The result of Briançon–Speder–Teissier is that the μ^*-sequence is constant in the family if and only if the family is Whitney equisingular.

A natural and long-standing question is: What happens in the nonisolated case? More precisely, we assume that we can stratify a family so that outside the parameter axis of the family we have a Whitney stratification and seek conditions that give an equivalence between a collection of topological invariants and Whitney equisingularity of the parameter axis. In some sense this was answered in [31] using the multiplicity of polar invariants. However, in many situations the number of invariants is very large. We would like a small number of topological or algebraic invariants, defined in a simple manner, that control and are controlled by the equisingularity of the family.

An important theorem of Gaffney and Gassler [12, Thm. 6.3] gives a partial result. They define the sequence χ^* as the Euler characteristic of the Milnor fibres that occur for the family. This is an obvious generalization of the μ^*-sequence, since the Euler characteristic of the Milnor fibre is determined by the Milnor number in the isolated singularity case. The constancy of this sequence does not seem to be sufficient to ensure Whitney equisingularity. Thus they define another sequence, called the relative polar multiplicities and denoted m_* (see Section 3 for a precise definition). In the case of isolated singularities, the constancy of μ^* in the family implies the constancy of m_* in the family.

Received July 30, 2009. Revision received June 2, 2010.