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Abelian Hurwitz–Hodge Integrals

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0. Introduction

0.1. Moduli of Covers

Let $\mathcal{M}_{g,n}$ be the moduli space of nonsingular, connected, genus-g curves over $\mathbb{C}$ with n distinct points. Let G be a finite group. Given an element $[C, p_1, \dots, p_n] \in \mathcal{M}_{g,n}$, we will consider principal G-bundles,

$$\begin{array}{ccc} G & \longrightarrow & P \\ & & \Big\downarrow{\scriptstyle \pi} \\ & & C \setminus \{p_1, \dots, p_n\}, \end{array} \tag{1}$$

over the punctured curve. Denote the G-action on the fibers of π by

$$\tau \colon G \times P \to P.$$

The monodromy defined by a positively oriented loop around the ith puncture determines a conjugacy class $\gamma_i \in \mathrm{Conj}(G)$. Let $\boldsymbol{\gamma} = (\gamma_1, \dots, \gamma_n)$ be the n-tuple of monodromies. The moduli space of covers $\mathcal{A}_{g,\gamma}(G)$ parameterizes G-bundles (1) with the prescribed monodromy conditions. There is a canonical morphism

$$\varepsilon \colon \mathcal{A}_{g,\gamma}(G) \to \mathcal{M}_{g,n}$$

obtained from the base of the G-bundle. Both $\mathcal{A}_{g,\gamma}(G)$ and $\mathcal{M}_{g,n}$ are nonsingular Deligne–Mumford stacks.

A compactification $\mathcal{A}_{g,\gamma}(G) \subset \overline{\mathcal{A}}_{g,\gamma}(G)$ by *admissible covers* was introduced by Harris and Mumford in [18]. An admissible cover

$$[\pi, \tau] \in \overline{\mathcal{A}}_{g,\gamma}(G)$$

is a degree-$|G|$ finite map of complete curves

$$\pi \colon D \to (C, p_1, \dots, p_n)$$

together with a G-action

$$\tau \colon G \times D \to D$$

on the fibers of π satisfying the following properties:

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