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On the Holomorphic Extension of CR Functions from Nongeneric CR Submanifolds of $\mathbb{C}^n$: The Positive Defect Case

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1. Introduction

1.1. Statement of Results

A real submanifold M of $\mathbb{C}^n$ is said to be CR if the dimension of $T_pM \cap iT_pM$ does not depend on p. Here M is a generic submanifold of $\mathbb{C}^n$ if, for any $p \in M$, $T_pM + JT_pM = T_p\mathbb{C}^n$. We say that a vector v in $\mathbb{C}^n$ is *complex transversal* to M at $p \in M$ if $v \notin \operatorname{span}_{\mathbb{C}} T_pM$. The question we address in this paper is the holomorphic extension of continuous CR functions on nongeneric CR submanifolds of $\mathbb{C}^n$ to wedges whose directions are complex transversal. In [4], we proved that any decomposable CR distribution admits a holomorphic extension to a complex transversal wedge. In this paper, we shall consider the case where CR functions (or distributions) are not decomposable. Define $\mathcal{O}_p^{\mathrm{CR}}$ to be the Sussmann manifold through p (the union of the CR orbits through p), which by [9] is a CR submanifold of M of *same CR dimension*. Assume the CR dimension (the complex dimension of $T_pM \cap iT_pM$) of M is k. Then the *defect of M at p* is said to be ℓ if the real dimension of $\mathcal{O}_p^{\mathrm{CR}}$ is $2k + \ell$. Our main result is the following theorem.

Theorem 1. *Let M be a smooth ($\mathcal{C}^\infty$) nongeneric CR submanifold of $\mathbb{C}^n$ of positive defect at some p. Then, for any v complex transversal to M at p and $\mathcal{U}$ a neighborhood of p, there exists a wedge $\mathcal{W}$ of direction v and with edge a neighborhood $\mathcal{U}' \subset \mathcal{U}$ of p in M such that any continuous CR function on $\mathcal{U}$ extends holomorphically to $\mathcal{W}$.*

1.2. Background

Theorem 1 generalizes results by Nagel and Rudin that imply a version of Theorem 1 in the totally real case. Let N be a smooth submanifold of the boundary of Ω, a strictly pseudoconvex domain in $\mathbb{C}^n$. If N is complex tangential ($TN \subset (T(\partial\Omega) \cap iT(\partial\Omega))$) then N is a peak interpolating set (see e.g. [7] or [8]). Given N, a totally real nongeneric submanifold of $\mathbb{C}^n$, one can easily construct Ω as just described and deduce Theorem 1 in the totally real case. As pointed out earlier, we first obtained Theorem 1 for the class of CR functions that are decomposable [4],

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