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Chow Motive of Fulton–MacPherson Configuration Spaces and Wonderful Compactifications

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1. Introduction

The purpose of this paper is to study the Chow groups and Chow motives of the so-called wonderful compactifications of an arrangement of subvarieties, in particular the Fulton–MacPherson configuration spaces.

All the varieties in the paper are over an algebraically closed field. Let Y be a nonsingular quasi-projective variety. Let $\mathcal{S}$ be an arrangement of subvarieties of Y (cf. Definition 2.2). Let $\mathcal{G}$ be a building set of $\mathcal{S}$, that is, a finite set of nonsingular subvarieties in $\mathcal{S}$ satisfying Definition 2.3. The wonderful compactification $Y_{\mathcal{G}}$ is constructed by blowing up Y along subvarieties in $\mathcal{G}$ successively (cf. Definition 2.5). There are different orders in which the blow-ups can be carried out; for example, we can blow up along the centers in any order that is compatible with the inclusion relation. There are many important examples of such compactifications: De Concini and Procesi's wonderful model of a subspace arrangement, the Fulton–MacPherson configuration spaces, the moduli space $\overline{\mathcal{M}}_{0,n}$ of stable rational curves with n marked points, and others. These spaces have many properties in common. Studying them with a uniform method gives us a better understanding of these spaces. In this paper, we study their Chow groups and Chow motives.

If we assume that Y is projective, then the Chow motive of $Y_{\mathcal{G}}$, denoted by $h(Y_{\mathcal{G}})$, can be decomposed canonically into a direct sum of the motive of Y and the twisted motives of the subvarieties in the arrangement (see Section 2.1 for a review of Chow motives). We will prove the following theorem, where the precise definitions of the set $M_{\mathcal{T}}$ and the subvarieties $Y_0\mathcal{T}$ of Y are given in Section 3.

Main Theorem (Theorems 3.1 and 3.2). *Let Y be a nonsingular quasi-projective variety, let $\mathcal{G}$ be a building set, and let $Y_{\mathcal{G}}$ be the wonderful compactification $Y_{\mathcal{G}}$. Then we have the Chow group decomposition*

$$A^*Y_{\mathcal{G}} = A^*Y \oplus \bigoplus_{\mathcal{T}} \bigoplus_{\underline{\mu} \in M_{\mathcal{T}}} A^{*-\|\underline{\mu}\|}(Y_0\mathcal{T}),$$

where $\mathcal{T}$ runs through all $\mathcal{G}$-nests. Moreover, when Y is projective we have the Chow motive decomposition

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