

# Functions of Vanishing Mean Oscillation Associated with Operators and Applications

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## 1. Introduction

Let  $L$  be the infinitesimal generator of an analytic semigroup on  $L^2(\mathbb{R}^n)$  with suitable upper bounds on its heat kernels, and suppose  $L$  has a bounded holomorphic functional calculus on  $L^2(\mathbb{R}^n)$ . In this paper, we introduce and develop a new function space  $\text{VMO}_L$  of vanishing mean oscillation associated with the operator  $L$ . Using the theory of tent spaces and the Littlewood–Paley theory, we prove that a Hardy space  $H_L^1$  of Auscher, Duong, and McIntosh introduced in [ADM<sub>Mc</sub>] is the dual of our new  $\text{VMO}_{L^*}$  in which  $L^*$  is the adjoint operator of  $L$ . We also give an equivalent characterization of the space  $\text{VMO}_L$  in the context of the theory of tent spaces.

A locally integrable function  $f$  on  $\mathbb{R}^n$  is said to be in  $\text{BMO}(\mathbb{R}^n)$ , the space of bounded mean oscillation, if

$$\|f\|_{\text{BMO}} = \sup_B |B|^{-1} \int_B |f(x) - f_B| dx < \infty, \quad (1.1)$$

where the supremum is taken over all balls  $B$  in  $\mathbb{R}^n$  and where  $f_B$  stands for the mean of  $f$  over  $B$ ; that is,

$$f_B = |B|^{-1} \int_B f(x) dx.$$

The quotient space of  $\text{BMO}(\mathbb{R}^n)$  with this seminorm over the constant functions is a Banach space. The space of BMO functions was introduced by John and Nirenberg [JN].

According to Sarason [Sa], a function  $f$  of  $\text{BMO}(\mathbb{R}^n)$  that satisfies the limiting condition

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