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A Characterization of Besov-type Spaces and Applications to Hankel-type Operators

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Introduction

Let $\mathbb{D}$ be the unit disc of the complex plane. Given a real number β, let

$$dA_\beta(z) = (1+\beta)(1-|z|^2)^\beta\, dA(z),$$

where dA is the normalized area measure on $\mathbb{D}$. For $\beta > -1$ and $0 < p < \infty$, the Bergman space A^p_β consists of all analytic functions in $L^p(dA_\beta) := L^p(\mathbb{D}, dA_\beta)$ with norm

$$\|f\|^p_{A^p_\beta} = \int_{\mathbb{D}} |f(z)|^p\, dA_\beta(z).$$

For $1 < p < \infty$ and $\alpha \le 1/2$, let $B_p(\alpha)$ be the Besov-type space of those analytic functions on the unit disc $\mathbb{D}$ for which

$$\|f\|^p_{\alpha,p} = |f(0)|^p + \int_{\mathbb{D}} |f'(z)|^p\, dA_{p,\alpha}(z) < \infty,$$

where

$$dA_{p,\alpha}(z) = (1-|z|^2)^{p-2+1-2\alpha}\, dA(z).$$

The space L^p_α is the space of smooth functions $u\colon \mathbb{D} \to \mathbb{C}$ for which

$$\|u\|^p_{\alpha,p} = |u(0)|^p + \int_{\mathbb{D}} |\nabla u(z)|^p\, dA_{p,\alpha}(z)$$

is finite. It is clear that $B_p(\alpha)$ is the subspace of all analytic functions in L^p_α. Note that the dual space of $B_p(\alpha)$ is isomorphic to $B_q(\alpha)$, where q is the conjugate exponent of p, under the pairing

$$\langle f, g\rangle_\alpha = f(0)\overline{g(0)} + \int_{\mathbb{D}} f'(z)\overline{g'(z)}(1-|z|^2)^{1-2\alpha}\, dA(z),$$

defined for $f \in B_p(\alpha)$ and $g \in B_q(\alpha)$. Note that, by Hölder's inequality, if $f \in B_p(\alpha)$ then $f' \in A^1_{1-2\alpha}$. So, using the reproducing formula for the Bergman space

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